Day 341 **IGCSE 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 22 is taken. Work out how many come from Group A.
(b) A different stratified sample has 27 from Group D. Work out the size of this sample.

Total = 60 + 60 + 140 + 180 = 440

(a) Group A: $\frac{60}{440} \times 22 = 3$

(b) Fraction sampled = $27 \div 180 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 440 = 66$

Answer: (a) 3 (b) 66

2 Error intervals

The number p is 63.98 correct to 2 decimal places.
Write down the error interval for p.

The rounding unit is 0.01, so the half-unit is 0.005
Lower bound = 63.98 − 0.005 = 63.975; upper bound = 63.98 + 0.005 = 63.985
Answer: $63.975 \leq p < 63.985$

3 f′(x) notation

$f(x) = 5x^3 - x^2 + 5$
(a) Find f′(x).
(b) Work out the value of f′(0).

(a) $5x^3 \to 15x^2$, $-x^2 \to -2x$
$f'(x) = 15x^2 - 2x$
(b) f′(0) = 0
Answer: (a) $f'(x) = 15x^2 - 2x$ (b) 0

4 Fractional indices

Given that x > 0, solve $x^{\frac{5}{3}} = 3125$

Raise both sides to the power $\frac{3}{5}$

$x = (3125)^{\frac{3}{5}}$
x = 125
Answer: x = 125

5 Trigonometric graphs: amplitude and period

For $0 \leq x \leq 360$, a graph has equation
$y = 2 \sin 4x° + 1$
Write down
(a) the least value of y
(b) the greatest value of y

(a) Least value = 1 − 2 = −1
(b) Greatest value = 2 + 1 = 3
Answer: (a) −1 (b) 3

6 Radius from chord and height of arc

A circular arc has a chord AB of length 36 cm.
The greatest distance between the chord and the arc is 6 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = $r - 6$

AM = $36 \div 2 = 18$. Pythagoras in triangle OMA: $r^2 = (r - 6)^2 + 18^2$

$r^2 = r^2 - 12r + 36 + 324$

$12r = 360$, so $r = 30$ cm

Answer: 30 cm

7 Given a stationary point, find k

The curve $y = x^3 + kx$ has a stationary point at $x = 3$.
(a) Find the value of k.
(b) Find the coordinates of the other stationary point.

$\frac{dy}{dx} = 3x^2 + k$

$x = 3$: $27 + k = 0$, so $k = -27$

$\frac{dy}{dx} = 3x^2 - 27 = 3(x - 3)(x + 3)$

Other root: $x = -3$

$y = 54$

Answer: (a) k = −27 (b) (−3, 54)

8 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and $a > 0$.
The curve passes through the points (1, 16) and (4, 128).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 16$ and $k\,a^4 = 128$

Divide: $a^3 = \frac{128}{16} = 8$

$a = 2$

From the first point: $k \times 2 = 16$, so $k = 8$

Answer: k = 8, a = 2

9 Increasing and decreasing with powers

$f(x) = 3x + \frac{3}{x}$ for $x > 0$

(a) Find the range of values of x for which f is a decreasing function.
(b) Write down the least value of f(x).

$f'(x) = 3 - 3x^{-2} = 3 - \frac{3}{x^2}$

f is decreasing when $f'(x) < 0$: $3 < \frac{3}{x^2}$, so $x^2 < 1$

With $x > 0$ this gives $0 < x < 1$

f decreases then increases, so the least value is at $x = 1$: $f(1) = 6$

Answer: (a) 0 < x < 1 (b) 6

10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 2. N is on OB with ON : NB = 1 : 2. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{3}{5}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{3}{5}\mathbf{a} = \frac{3}{5}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{3}{5}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{3}t\mathbf{b}$

Equate: $\frac{3}{5}s = 1 - t$ and $1 - s = \frac{1}{3}t$

Solving: $s = \frac{5}{6}$ and $t = \frac{1}{2}$

OX = $\frac{1}{2}\mathbf{a} + \frac{1}{6}\mathbf{b}$

Answer: (a) $\frac{3}{5}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{2}\mathbf{a} + \frac{1}{6}\mathbf{b}$