Day 340 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 19:55 (London time).
It takes 11 hours 30 minutes.
When it is 12:00 in London it is 17:00 in Delhi.
At what time, in Delhi time, does the flight land? Say if it is on a different day.

Arrival in London time = 19:55 + 11 h 30 min = 07:25 the next day
Delhi is 5 hours ahead of London, so add 5 hours
12:25 the next day
Answer: 12:25 the next day

2 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 23 is taken. Work out how many come from Group D.
(b) A different stratified sample has 15 from Group D. Work out the size of this sample.

Total = 60 + 40 + 80 + 50 = 230

(a) Group D: $\frac{50}{230} \times 23 = 5$

(b) Fraction sampled = $15 \div 50 = \frac{3}{10}$, so sample size = $\frac{3}{10} \times 230 = 69$

Answer: (a) 5 (b) 69

3 Direct variation

F is directly proportional to s.
When s = 10, F = 150.
(a) Find a formula for F in terms of s.
(b) Find the value of s when F = 75.

F = ks
150 = k × 10 so k = 15
F = 15s
75 = 15s so s = 5
s = 5
Answer: (a) F = 15s (b) s = 5

4 Simultaneous equations

Solve the simultaneous equations
2x + 5y = 26
4x − 3y = −26

To match the x terms: (1) × 2 gives 4x + 10y = 52
Subtract the equations to eliminate x: 13y = 78
y = 6
Substitute y = 6 into (1): 2x + 30 = 26
2x = −4, so x = −2
Answer: x = −2, y = 6

5 Greatest and least values on an interval

$f(x) = x^3 - 12x - 2$ for $-3 \le x \le 3$.
Find the greatest value and the least value of f(x) on this interval.

$f'(x) = 3x^2 - 12 = 0$ gives x = −2 or x = 2 (both in the interval)
f(−3) = 7, f(−2) = 14, f(2) = −18, f(3) = −11
Compare the turning points and both end values
Answer: Greatest 14, least −18

6 Venn diagram with three sets: algebra

The Venn diagram shows information about 39 pupils.
A: play football B: play netball C: play rugby
(a) Find the value of x.
(b) Find n(A) and n(B ∪ C)′.

(a) 3x + 24 = 39
3x = 15, so x = 5
(b) Substitute x = 5 to fill in every region
n(A) = 5 + 2 + 5 + 7 = 19
n(B ∪ C)′ = 7 + 3 = 10 (A only plus outside)
Answer: (a) x = 5 (b) n(A) = 19, n(B ∪ C)′ = 10

7 Rate of change of area or volume

A cylinder has radius r cm and fixed height 10 cm. Its volume is $V = 10\pi r^2$ cm^3.
(a) Find $\frac{dV}{dr}$ when r = 8.
(b) Find the value of r when $\frac{dV}{dr} = 160\pi$.

$\frac{dV}{dr} = 20\pi r$
When r = 8: $\frac{dV}{dr} = 160\pi$
$20\pi r = 160\pi$, so 20r = 160 and r = 8 (lengths are positive)
Answer: (a) 160π (b) r = 8

8 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 65$
$x + y = -11$

From the second equation, $y = -x - 11$
Substitute into the first equation: $x^2 + (-x - 11)^2 = 65$
Expand: $2x^2 + 22x + 56 = 0$
Divide by 2: $x^2 + 11x + 28 = 0$
Factorise: $(x + 7)(x + 4) = 0$, so $x = -7$ or $x = -4$
$y = -x - 11$ gives $y = -4$ and $y = -7$
Answer: x = −7, y = −4 and x = −4, y = −7

9 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M. (Calculator)
(a) Work out AC as a simplified surd.
(b) Work out the angle between VA and the base, to 1 d.p.
The diagram is not drawn accurately.

(a) $AC^2 = 8^2 + 8^2 = 128$
$AC = \sqrt{128} = 8\sqrt{2}$ cm
(b) AM = AC ÷ 2 = $4\sqrt{2}$ cm. The angle is VAM.
$\tan(VAM) = \frac{VM}{AM} = \frac{2}{\sqrt{32}}$
angle VAM = 19.5° (1 d.p.)
Answer: (a) $8\sqrt{2}$ cm (b) 19.5°

10 Where two tangents meet

The curve C has equation $y = -2x^2 + 3x - 5$.
The points A and B on C have x coordinates -2 and 0.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = -4x + 3$. At A: gradient 11, A = $(-2, -19)$
Tangent at A: $y = 11x + 3$. At B: gradient 3, B = $(0, -5)$
Tangent at B: $y = 3x - 5$
Equate: $11x + 3 = 3x - 5$, so $x = -1$ and $y = -8$
Answer: T $(-1, -8)$