Day 34 **IGCSE 10-a-day: Model Solutions**

1 Rational and irrational numbers

Here are six numbers.

$3 + \sqrt{12}, \sqrt{3} \times \sqrt{5}, \sqrt{50}, \sqrt{\frac{9}{16}}, \sqrt{2} + \sqrt{8}, \frac{\sqrt{18}}{\sqrt{2}}$

Write down the irrational numbers.

Simplify each number:

$3 + \sqrt{12} = 3 + 2\sqrt{3}$; $\sqrt{3} \times \sqrt{5} = \sqrt{15}$; $\sqrt{50} = 5\sqrt{2}$

$\sqrt{\frac{9}{16}} = \frac{3}{4}$; $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$; $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$

Answer: $3 + \sqrt{12}$, $\sqrt{3} \times \sqrt{5}$, $\sqrt{50}$, $\sqrt{2} + \sqrt{8}$

2 Second derivative

$y = 3x^3 + 6x^2 + 4x$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Work out the value of $\frac{d^2y}{dx^2}$ when $x = 2$.

$\frac{dy}{dx} = 9x^2 + 12x + 4$

$\frac{d^2y}{dx^2} = 18x + 12$

$x = 2$: $\frac{d^2y}{dx^2} = 48$

Answer: (a) $\frac{dy}{dx} = 9x^2 + 12x + 4$, $\frac{d^2y}{dx^2} = 18x + 12$ (b) 48

3 Conditional probability from a two-way table

The table shows how 133 students travel to school.

A person who chose Bus is picked at random. Work out the probability that this person is in the group "Girls".

Total who chose Bus = 23 + 19 = 42

Girls who chose Bus = 19

Probability = $\frac{19}{42}$

Answer: $\frac{19}{42}$

4 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 24°.

(a) Work out the size of angle ACB.

(b) Work out the size of angle COB.

The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°

(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 24°

Angle COB is an exterior angle of triangle OAC: 24 + 24 = 48°

(or: the angle at the centre is twice the angle at the circumference)

Answer: (a) 90° (b) 48°

5 Forming and solving quadratic equations

A rectangle has length (3x + 6) cm and width (x + 3) cm.

The area of the rectangle is 330 cm^2.

(a) Show that $3x^2 + 15x - 312 = 0$

(b) Work out the perimeter of the rectangle.

(a) $(3x + 6)(x + 3) = 330$, so $3x^2 + 15x + 18 = 330$

$3x^2 + 15x - 312 = 0$

(b) $3x^2 + 15x - 312 = 0$: $3(x - 8)(x + 13) = 0$

$x = 8$ ($x = -13$ is not possible as lengths are positive)

Length = 30 cm and width = 11 cm, so perimeter = $2 \times (30 + 11) = 82$ cm

Answer: (b) 82 cm

6 Three-part ratios

Sofia, Zane and Uma share some money.

Sofia : Zane = 7 : 2

Zane : Uma = 9 : 3

(a) Work out the ratio Sofia : Zane : Uma.

(b) They share £145 altogether. How much money does Sofia get?

(a) Make the Zane part the same: 7 : 2 = 63 : 18 and 9 : 3 = 18 : 6

Sofia : Zane : Uma = 63 : 18 : 6 = 21 : 6 : 2

(b) Total parts = 29, so 1 part = 145 ÷ 29 = 5

Sofia gets $21 \times 5 = £105$

Answer: (a) 21 : 6 : 2 (b) £105

7 Gradient function from a given point

A curve has equation $y = 2x^3 + 9x^2 + 1$.

The point A (3, k) lies on the curve. Work out the value of k and the gradient of the curve at A.

Substitute $x = 3$ into the equation of the curve

$y = 136$ when $x = 3$, so $k = 136$

$\frac{dy}{dx} = 6x^2 + 18x$

At A: gradient = 108

Answer: k = 136, gradient = 108

8 Fractions of fractions

Dina read $\frac{6}{7}$ of a book on Saturday.

On Sunday she read $\frac{1}{4}$ of the pages that were left.

She read 11 pages on Sunday.

How many pages are in the book?

The second amount is $\frac{1}{4}$ of $\frac{1}{7}$, which is $\frac{1}{28}$ of the total

$\frac{1}{28}$ of the total is 11

Total = $11 \div \frac{1}{28} = 308$

Answer: 308 pages

9 Area of triangle from a tangent

The tangent to the curve $y = x^2 + x - 4$ at the point where $x = -1$ meets the axes at A and B.
Work out the area of triangle OAB, where O is the origin.

$y = -4$ and $\frac{dy}{dx} = 2x + 1$, so the gradient is -1

Tangent: $y = -x - 5$

y-axis: $(0, -5)$. x-axis: $0 = -x - 5$ so $x = -5$

Area $= \frac{1}{2} \times 5 \times 5$

$= \frac{25}{2}$

Answer: $\frac{25}{2}$ square units

10 Intersecting lines in a triangle

$\mathbf{OA} = \mathbf{a}$, $\mathbf{OB} = \mathbf{b}$, $\mathbf{OP} = \frac{1}{3}\mathbf{a}$ and $\mathbf{OQ} = \frac{3}{4}\mathbf{b}$.

AQ and BP meet at X.
(a) Find $\mathbf{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find the ratio AX : XQ.

Let $\mathbf{AX} = s\mathbf{AQ}$. Then $\mathbf{OX} = \mathbf{a} + s(-\mathbf{a} + \frac{3}{4}\mathbf{b}) = (1 - s)\mathbf{a} + \frac{3}{4}s\mathbf{b}$

Let $\mathbf{BX} = t\mathbf{BP}$. Then $\mathbf{OX} = \mathbf{b} + t(\frac{1}{3}\mathbf{a} - \mathbf{b}) = \frac{1}{3}t\mathbf{a} + (1 - t)\mathbf{b}$

Compare coefficients: $1 - s = \frac{1}{3}t$ and $\frac{3}{4}s = 1 - t$

Solving gives $s = \frac{8}{9}$ and $t = \frac{1}{3}$

(a) $\mathbf{OX} = \frac{1}{9}\mathbf{a} + \frac{2}{3}\mathbf{b}$

(b) $AX = \frac{8}{9} \times AQ$, so $AX : XQ = \frac{8}{9} : \frac{1}{9} = 8 : 1$

Answer: (a) $\frac{1}{9}\mathbf{a} + \frac{2}{3}\mathbf{b}$ (b) 8 : 1