Day 339 **IGCSE 10-a-day: Model Solutions**

1 Ratio and expressions

x : y = 4 : 9
Work out the ratio (6x − 1y) : (3x + 6y)
Give your answer in its simplest form.

x : y = 4 : 9, so let x = 4t and y = 9t
6(4t) − 1(9t) = 15t
3(4t) + 6(9t) = 66t
15t : 66t = 5 : 22
Answer: 5 : 22

2 Time zones

A flight leaves London at 06:45 (London time).
It takes 5 hours 10 minutes.
When it is 12:00 in London it is 17:00 in Delhi.
At what time, in Delhi time, does the flight land? Say if it is on a different day.

Arrival in London time = 06:45 + 5 h 10 min = 11:55
Delhi is 5 hours ahead of London, so add 5 hours
16:55
Answer: 16:55 the same day

3 Equations with fractions

Solve $\frac{x-2}{6} - \frac{x-6}{2} = 4$

Multiply every term by 6: (x − 2) − 3(x − 6) = 24
Expand: −2x + 16 = 24
−2x = 8
x = −4
Answer: x = −4

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 66°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: 360 − 90 − 90 − 66 = 114°
(b) The angle at the circumference is half the angle at the centre: 114 ÷ 2 = 57°
Answer: (a) 114° (b) 57°

5 Negative powers

$y = -3x^2 + 5x + \frac{8}{x^2}$

Find $\frac{dy}{dx}$.

Write negative powers as indices: $y = -3x^2 + 5x + 8x^{-2}$
$-3x^2 \to -6x$, $5x \to 5$, $8x^{-2} \to -16x^{-3}$
$\frac{dy}{dx} = -6x + 5 - 16x^{-3}$
Answer: $\frac{dy}{dx} = -6x + 5 - \frac{16}{x^3}$

6 Exponential growth and decay

A population of fish starts at 3,000 and grows by 20% each year.
After how many whole years will the population first be at least double its starting value? (Calculator)

Multiplier = 1.2, double is 6,000

After 3 years: $3000 \times 1.2^3 = 5184.0$

After 4 years: $3000 \times 1.2^4 = 6220.8$

Answer: 4 years

7 Depreciation

A van is bought for £12,000.
Its value falls by 15% each year.
Work out its value after 3 years. (Calculator)

Multiplier = $1 - 15 \div 100 = 0.85$

Value = 12000×0.85^3

= £7,369.50

Answer: £7,369.50

8 Increasing range for y = ax + b/x

$y = 4x + \frac{36}{x}$

Find the values of x for which y is increasing.

$\frac{dy}{dx} = 4 - \frac{36}{x^2}$

$\frac{dy}{dx} > 0$: $4 > \frac{36}{x^2}$

$x^2 > 9$ (x^2 is positive, so multiply through)

$x < -3$ or $x > 3$

Answer: $x < -3$ or $x > 3$

9 Venn diagram with three sets: algebra

The Venn diagram shows information about 40 shoppers.
A: bought fruit B: bought bread C: bought milk
(a) Find the value of x.
(b) Find n(A) and $n(B \cup C)'$.

(a) $4x + 24 = 40$

$4x = 16$, so $x = 4$

(b) Substitute $x = 4$ to fill in every region

$n(A) = 6 + 3 + 3 + 6 = 18$

$n(B \cup C)' = 6 + 5 = 11$ (A only plus outside)

Answer: (a) $x = 4$ (b) $n(A) = 18$, $n(B \cup C)' = 11$

10 Wire cut in two

A piece of wire 112 cm long is cut into two pieces. One piece is bent to form a square of side x cm. The other piece is bent to form a rectangle whose length is three times its width.

(a) Show that the total area, A cm^2, enclosed by the two shapes is $A = x^2 + \frac{3(112 - 4x)^2}{64}$.

(b) Find the value of x for which A is least, and work out this least area.

Rectangle perimeter $112 - 4x = 8w$, so $w = \frac{112 - 4x}{8}$ and its area is $3w^2 = \frac{3(112 - 4x)^2}{64}$

$A = x^2 + \frac{3(112 - 4x)^2}{64} = \frac{7}{4}x^2 - 42x + 588$

$\frac{dA}{dx} = \frac{7}{2}x - 42 = 0$, so $x = 12$. $\frac{d^2A}{dx^2} = \frac{7}{2} > 0$, so a minimum

Square: 144, rectangle: $24 \times 8 = 192$, so $A = 336$

Answer: $x = 12$, least area 336 cm^2