Day 338 **IGCSE 10-a-day: Model Solutions**

1 Gradient at a point

Work out the gradient of the curve $y = 4x^4 + 4x^3 - 7x^2$ at the point where $x = 0$.

$\frac{dy}{dx} = 16x^3 + 12x^2 - 14x$

When $x = 0$: gradient $= 0$

Answer: 0

2 Percentage change

The population of a village increased by 10% in 2023.
In 2024 the population increased by a further 10%.
Work out the overall percentage increase over the two years.

Multipliers: 1.1 and 1.1

Overall multiplier $= 1.1 \times 1.1 = 1.21$

$1.21 - 1 = 0.21$, so an overall increase of 21%

Answer: 21% increase

3 Inverse proportion in context

10 painters can paint a hall in 30 days.
After 2 days, 2 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work $= 10 \times 30 = 300$ painter-days

Work done $= 10 \times 2 = 20$, so work left $= 280$

Workers left $= 10 - 2 = 8$

Time $= 280 \div 8 = 35$ days

Answer: 35 days

4 Solve a linear inequality

Solve $\frac{x - 4}{4} \geq \frac{2 + 2x}{5}$

Multiply by 20: $5(x - 4) \geq 4(2 + 2x)$

Expand and collect: $-3x \geq 28$

Divide by -3 and reverse the inequality sign: $x \leq -\frac{28}{3}$

Answer: $x \leq -\frac{28}{3}$

5 Changing the subject

Make r the subject of $P = 6r^3 + 14$

Subtract 14: $P - 14 = 6r^3$

Divide by 6: $r^3 = \frac{P - 14}{6}$

Cube root: $r = \sqrt[3]{\frac{P - 14}{6}}$

Answer: $r = \sqrt[3]{\frac{P - 14}{6}}$

6 Point with a given gradient (quadratic)

A curve has equation $y = 2x^2 + 8x - 3$.
The gradient at the point P on the curve is 12.
Find the coordinates of P.

$\frac{dy}{dx} = 4x + 8$

$4x + 8 = 12$, so $4x = 4$

$x = 1$

$y = 7$

Answer: (1, 7)

7 Cumulative frequency graphs

The cumulative frequency graph gives information about the marks of 100 students in a test.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 100 had a mark greater than 55.

(a) Median at cumulative frequency 50, so median = 40
Lower quartile at 25, so LQ = 30. Upper quartile at 75, so UQ = 50
Interquartile range = 50 − 30 = 20
(b) At 55, cumulative frequency = 82
Number greater = 100 − 82 = 18
Answer: (a) median 40, IQR 20 (b) 18

8 Venn diagram with three sets: algebra

The Venn diagram shows information about 41 students.
A: study Art B: study Music C: study Drama
(a) Find the value of x.
(b) Find n(A) and n(B ∪ C)′.

(a) $4x + 17 = 41$
$4x = 24$, so $x = 6$
(b) Substitute x = 6 to fill in every region
n(A) = 2 + 6 + 3 + 2 = 13
n(B ∪ C)′ = 2 + 4 = 6 (A only plus outside)
Answer: (a) x = 6 (b) n(A) = 13, n(B ∪ C)′ = 6

9 Stationary point of y = ax + b/√x

A curve has equation $y = 4x + \frac{64}{\sqrt{x}}$ for $x > 0$.

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 4 - 32x^{-3/2}$

$\frac{dy}{dx} = 0$: $32x^{-3/2} = 4$, so $x^{3/2} = 8$

$x = 8^{2/3} = 4$, and $y = 48$

$\frac{d^2y}{dx^2} = 48x^{-5/2}$

At x = 4, $\frac{d^2y}{dx^2} = \frac{3}{2}$, which is positive, so a minimum

Answer: (4, 48), minimum

10 Tangent to a circle

A circle has equation $x^2 + y^2 = 625$.
The point P(24, 7) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $\frac{7}{24}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div \frac{7}{24} = -\frac{24}{7}$

$y - 7 = (-\frac{24}{7})(x - 24)$

Multiply out and rearrange: $24x + 7y = 625$
(b) On the x-axis y = 0, so $24x = 625$
$x = \frac{625}{24}$, so T is $(\frac{625}{24}, 0)$

Answer: (a) 24x + 7y = 625 (b) T = $(\frac{625}{24}, 0)$