Day 337 **IGCSE 10-a-day: Model Solutions**

1 Parallelogram and midpoint with vectors

ABCD is a parallelogram.
A = (−1, 4), B = (−3, 1) and D = (−3, 0).
Find the coordinates of C and of the point where the diagonals meet.

$\mathbf{BC} = \mathbf{AD} = \text{D} - \text{A} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

C = B + **AD** = (−5, −3)

The diagonals bisect each other, so they meet at the midpoint of AC

$\left(\frac{-1 + (-5)}{2}, \frac{4 + (-3)}{2}\right) = \left(-3, \frac{1}{2}\right)$

Answer: C = (−5, −3), diagonals meet at $\left(-3, \frac{1}{2}\right)$

2 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where
$s = 2t^3 - 9t^2 + 8t$ $(t > 0)$
Find (a) the velocity of P, (b) the acceleration of P, when t = 2.

$v = \frac{ds}{dt} = 6t^2 - 18t + 8$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 12t - 18$

When t = 2: v = −4 and a = 6

Answer: (a) −4 m/s (b) 6 m/s^2

3 Percentage change

The population of a village decreased by 5% in 2023.
In 2024 the population decreased by a further 5%.
Work out the overall percentage decrease over the two years.

Multipliers: 0.95 and 0.95
Overall multiplier = 0.95 × 0.95 = 0.9025
0.9025 − 1 = −0.0975, so an overall decrease of 9.75%
Answer: 9.75% decrease

4 Inverse proportion in context

6 painters can paint a hall in 8 days.
After 5 days, 5 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 6 × 8 = 48 painter-days
Work done = 6 × 5 = 30, so work left = 18
Workers left = 6 − 5 = 1
Time = 18 ÷ 1 = 18 days
Answer: 18 days

5 Iteration

Use the iteration formula $x_{n+1} = \sqrt{2x_n + 8}$ with $x_0 = 3$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt{2 \times 3 + 8} = 3.74166...$

$x_2 = \sqrt{2 \times x_1 + 8} = 3.93488...$

$x_3 = \sqrt{2 \times x_2 + 8} = 3.98369...$

Rounded: $x_1 = 3.742$, $x_2 = 3.935$, $x_3 = 3.984$

Answer: $x_1 = 3.742$, $x_2 = 3.935$, $x_3 = 3.984$

6 Find constants from a gradient

A curve has equation $y = ax^2 + \frac{b}{x}$, where a and b are constants.

The point (1, 1) lies on the curve and the gradient of the curve at this point is 8.
Find the values of a and b.

$\frac{dy}{dx} = 2ax - bx^{-2} = 2ax - \frac{b}{x^2}$

Point on curve: $a + b = 1$. Gradient: $2a - b = 8$
Solving simultaneously gives $a = 3$ and $b = -2$
Answer: $a = 3$, $b = -2$

7 Bearings with the sine and cosine rules

A walker leaves a camp P and travels 35 km on a bearing of 285° to Q. (Calculator)
It then travels 64 km on a bearing of 220° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is $285° - 180° = 105°$
Angle PQR = 115° (the difference between the bearings 105° and 220°)
$PR^2 = 35^2 + 64^2 - 2 \times 35 \times 64 \times \cos 115° = 7214.3$, so PR = 84.9 km

(b) $\cos(QPR) = \frac{35^2 + PR^2 - 64^2}{2 \times 35 \times PR}$, so angle QPR = 43.1°
R is anticlockwise of the direction PQ, so the bearing is $285° - 43° = 242°$
Answer: (a) 84.9 km (b) 242°

8 Express a vector in terms of two vectors

$\mathbf{a} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -8 \\ -9 \end{pmatrix}$.
$\mathbf{c} = m\mathbf{a} + n\mathbf{b}$.
Find the value of m and the value of n.

$-4m + 4n = -8$ and $-3m + 2n = -9$
Multiply the first by 2 and the second by 4: $-8m + 8n = -16$ and $-12m + 8n = -36$
Subtract: $4m = 20$, so $m = 5$
Substitute back: $n = 3$
Answer: $m = 5$, $n = 3$

9 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 1. N is on OB with ON : NB = 1 : 3. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{3}{4}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{3}{4}\mathbf{a} = \frac{3}{4}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{3}{4}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{4}t\mathbf{b}$

Equate: $\frac{3}{4}s = 1 - t$ and $1 - s = \frac{1}{4}t$

Solving: $s = \frac{12}{13}$ and $t = \frac{4}{13}$

OX = $\frac{9}{13}\mathbf{a} + \frac{1}{13}\mathbf{b}$

Answer: (a) $\frac{3}{4}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{9}{13}\mathbf{a} + \frac{1}{13}\mathbf{b}$

10 Three stationary points of a quartic

$y = 3x^4 - 24x^2 + 4$

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 12x^3 - 48x = 12x(x - 2)(x + 2)$

$\frac{dy}{dx} = 0$ gives $x = -2$, $x = 0$ or $x = 2$

Points: $(-2, -44)$, $(0, 4)$, $(2, -44)$

$\frac{d^2y}{dx^2} = 36x^2 - 48$

$x = -2$: $\frac{d^2y}{dx^2} = 96$; $x = 0$: $\frac{d^2y}{dx^2} = -48$; $x = 2$: $\frac{d^2y}{dx^2} = 96$

So $(-2, -44)$ is a minimum, $(0, 4)$ is a maximum, $(2, -44)$ is a minimum

Answer: $(-2, -44)$ minimum; $(0, 4)$ maximum; $(2, -44)$ minimum