Day 336 **IGCSE 10-a-day: Model Solutions**

1 Capture and recapture

40 fish in a lake are caught, tagged and put back.
Later, a second sample of 20 fish is caught. 4 of these are tagged.
(a) Work out an estimate for the number of fish in a lake.
(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 4 out of 20

This should match the proportion tagged in the whole population: $\frac{4}{20} = \frac{40}{N}$

$N = 40 \times 20 \div 4 = 200$

(b) No fish are born, die or leave between the samples, and the fish mix evenly

Answer: (a) 200 (b) The population stays the same between samples

2 Product of brackets: tangent gradient

$y = x(x + 3)(3x - 1)$
Work out the gradient of the curve at the point where $x = 0$.

Expand: $y = 3x^3 + 8x^2 - 3x$

$\frac{dy}{dx} = 9x^2 + 16x - 3$

$x = 0$: $\frac{dy}{dx} = -3$

Answer: -3

3 Inverse of a rational function

$f(x) = \frac{5x - 5}{4x + 3}$, $x \neq -\frac{3}{4}$.

Find $f^{-1}(x)$.

$y = \frac{5x - 5}{4x + 3}$, so $y(4x + 3) = 5x - 5$

$4xy + 3y = 5x - 5$

$4xy - 5x = -5 - 3y$

$x(4y - 5) = -5 - 3y$, so $x = \frac{-5 - 3y}{4y - 5}$

$f^{-1}(x) = \frac{-5 - 3x}{4x - 5}$

Answer: $\mathbf{f^{-1}(x) = \frac{-5 - 3x}{4x - 5}}$

4 Equation of a tangent

A curve has equation $y = 2x^2 - 5x + 5$.
Find the equation of the tangent to the curve at the point where $x = -3$.

When $x = -3$, $y = 38$

$\frac{dy}{dx} = 4x - 5$, so the gradient at $x = -3$ is -17

$y - 38 = -17(x + 3)$

$y = -17x - 13$

Answer: $y = -17x - 13$

5 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{-4-0}{0-(-2)} = -2$

Perpendicular gradient = $-1 \div (-2) = \frac{1}{2}$

M passes through B(−2, 0): $0 = \frac{1}{2} \times (-2) + c$, so c = 1

$y = \frac{1}{2}x + 1$

Answer: $y = \frac{1}{2}x + 1$

6 Trigonometric graphs

The diagram shows the graph of y = a cos bx, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when x = 45°.

(a) The maximum value is 2 and the minimum is −2, so a = 2
The graph completes 2 full cycles between 0° and 360°, so b = 2
(b) y = 2 cos (2 × 45°) = 2 cos 90°
cos 90° = 0, so y = 0
Answer: (a) a = 2, b = 2 (b) y = 0

7 Greatest and least values on an interval

$f(x) = 2x^3 - 6x^2 - 3$ for $-2 \le x \le 4$.
Find the greatest value and the least value of f(x) on this interval.

$f'(x) = 6x^2 - 12x = 0$ gives x = 0 or x = 2 (both in the interval)
f(−2) = −43, f(0) = −3, f(2) = −11, f(4) = 29
Compare the turning points and both end values
Answer: Greatest 29, least −43

8 Ratio and money transferred

Evan and Callum share some money in the ratio 8 : 9.
Evan gives Callum £26.
The ratio of their money is now 3 : 5.
How much money do they have altogether?

Let the amounts be 8k and 9k, so the total is 17k

Evan now has 8k − 26, which is $\frac{3}{8}$ of the total

8(8k − 26) = 3 × 17k
64k − 208 = 51k, so 13k = 208 and k = 16
Total = 17 × 16 = £272
Answer: £272

9 Bounds of speed, density and pressure

A metal block has a mass of 1200 g, correct to the nearest 100.
Its volume is 95 cm^3, correct to the nearest whole number.
Work out the lower bound of the density.
Give your answer correct to 3 significant figures. (Calculator)

1150 ≤ mass < 1250; 94.5 ≤ volume < 95.5
Lower bound of density = smallest mass ÷ biggest volume = 1150 ÷ 95.5
= 12.0419... = 12.0 (3 s.f.)
Answer: 12.0 g/cm^3

10 Fibonacci-type sequences

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$.

$u_1 = 5$ and $u_2 = x$.

(a) Show that $u_5 = 3x + 10$.

(b) Given that $u_5 = 19$, find x and hence work out u_8.

(a) $u_3 = 5 + x$, $u_4 = u_3 + u_2 = 2x + 5$, $u_5 = u_4 + u_3 = 3x + 10$

(b) $3x + 10 = 19$ so $3x = 9$ and $x = 3$

Terms: 5, 3, 8, 11, 19, 30, 49, 79

$u_8 = 79$

Answer: $x = 3$, $u_8 = 79$