Day 335 **IGCSE 10-a-day: Model Solutions**

1 Changing the subject

Make r the subject of $P = 3r^3 + 13$

Subtract 13: $P - 13 = 3r^3$

Divide by 3: $r^3 = \frac{P - 13}{3}$

Cube root: $r = \sqrt[3]{\frac{P - 13}{3}}$

Answer: $r = \sqrt[3]{\frac{P - 13}{3}}$

2 Rate of cooling or growth

The temperature, T °C, of a metal bar t minutes after it is removed from a heat source is modelled by

$T = 25 + \frac{64}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when t = 2.

(b) Find the value of t when the temperature is falling at 16 °C per minute.

$T = 25 + 64t^{-1}$, so $\frac{dT}{dt} = -64t^{-2}$

When t = 2: $\frac{dT}{dt} = -16$

$-\frac{64}{t^2} = -16$, so $t^2 = 4$ and t = 2

Answer: (a) −16 °C/min (b) t = 2

3 Solving AX = B

$A = \begin{pmatrix} -4 & 6 \\ -2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 20 \\ 22 \end{pmatrix}$.

AX = B, where X is a 2 × 1 matrix.

Find X.

$X = A^{-1}B$ (multiply both sides on the left by A^{-1})

$A^{-1} = -\frac{1}{8}\begin{pmatrix} 5 & -6 \\ 2 & -4 \end{pmatrix}$

$X = -\frac{1}{8}\begin{pmatrix} 5 & -6 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} 20 \\ 22 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

Answer: $X = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

4 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 3 cm, DB = 9 cm and the area of triangle ADE is 18 cm^2.

Work out the area of the trapezium DBCE.

The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 3 + 9 = 12 cm

Length scale factor = 12 ÷ 3 = 4

Area scale factor = $(4)^2 = 16$

Area of triangle ABC = 18 × 16 = 288 cm^2

Trapezium = 288 − 18 = 270 cm^2

Answer: 270 cm^2

5 Maximum speed from velocity

A particle moves in a straight line. Its velocity, v m/s, at time t seconds is given by

$v = 50t - 5t^2$ for $0 \leq t \leq 10$

Find the time at which the speed of the particle is greatest, and this greatest speed.

$a = \frac{dv}{dt} = -10t + 50$

Greatest speed when $a = 0$: $-10t + 50 = 0$, so $t = 5$

$\frac{d^2v}{dt^2} = -10 < 0$, so this is a maximum

$v = 50 \times 5 - 5 \times 5^2 = 125$

Answer: t = 5 s, greatest speed 125 m/s

6 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.

(a) Work out the interquartile range for each.

(b) Compare the distributions of A and B.

(a) IQR of A = 55 − 40 = 15

IQR of B = 40 − 20 = 20

(b) Class A has the higher median (50 compared with 30), so on average Class A scored higher

Class A has the smaller interquartile range (15 compared with 20), so its marks are more consistent

Answer: (a) A: 15, B: 20 (b) A has the higher median, A has the smaller IQR

7 Rational and irrational numbers

Here are six numbers.

$\sqrt{2} + \sqrt{8}$, $\frac{\sqrt{18}}{\sqrt{2}}$, $(1 + \sqrt{2})^2$, $\sqrt{0.25}$, $(\sqrt{3})^2$, $0.\dot{4}$

Write down the rational numbers.

Simplify each number:

$\sqrt{2} + \sqrt{8} = 3\sqrt{2}$; $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$; $(1 + \sqrt{2})^2 = 3 + 2\sqrt{2}$

$\sqrt{0.25} = 0.5$; $(\sqrt{3})^2 = 3$; $0.\dot{4} = \frac{4}{9}$

Answer: $\frac{\sqrt{18}}{\sqrt{2}}$, $\sqrt{0.25}$, $(\sqrt{3})^2$, $0.\dot{4}$

8 Rearranging when the subject appears twice

Make r the subject of $P = \frac{-4r - 4}{r - 4}$

Multiply by $r - 4$: $P(r - 4) = -4r - 4$

Expand: $rP - 4P = -4r - 4$

Collect the r terms: $rP + 4r = -4 + 4P$

Factorise: $r(P + 4) = 4P - 4$

$r = \frac{4P - 4}{P + 4}$

Answer: $r = \frac{4P - 4}{P + 4}$

9 Tangent parallel to a chord

The diagram shows part of the curve $y = 2x^2 - x + 6$. A and B are the points on the curve with x coordinates -1 and 3. C is the point on the curve between A and B at which the tangent is parallel to AB. Find the coordinates of C.

A = $(-1, 9)$ and B = $(3, 21)$

Gradient of AB = $\frac{21 - 9}{3 - (-1)} = 3$

$\frac{dy}{dx} = 4x - 1 = 3$, so $x = 1$

$y = 7$

Answer: C (1, 7)

10 Show that: trigonometric identities

Show that $\frac{1 - \cos^2 x}{\cos^2 x} \equiv \tan^2 x$.

Use $\sin^2 + \cos^2 \equiv 1$, so $1 - \cos^2 x \equiv \sin^2 x$

$\frac{\sin^2 x}{\cos^2 x} \equiv (\sin x \div \cos x)^2 = \tan^2 x$

Since tan = sin ÷ cos, this equals $\tan^2 x$

Answer: Shown