Day 334 **IGCSE 10-a-day: Model Solutions**

1 Negative powers

Differentiate $-6x^4 + \frac{5}{x}$ with respect to x.

Write negative powers as indices: $y = -6x^4 + 5x^{-1}$

$-6x^4 \to -24x^3$, $5x^{-1} \to -5x^{-2}$

$\frac{dy}{dx} = -24x^3 - 5x^{-2}$

Answer: $\mathbf{\frac{dy}{dx} = -24x^3 - \frac{5}{x^2}}$

2 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 1. N is on AB with AN : NB = 1 : 3.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Show that MN is parallel to OB.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{4}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{4}(\mathbf{b} - \mathbf{a}) = \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

(b) OM = $\frac{3}{4}\mathbf{a}$

MN = ON − OM = $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b} - \frac{3}{4}\mathbf{a} = \frac{1}{4}\mathbf{b}$

MN = $\frac{1}{4}\mathbf{b}$, which is a multiple of OB, so MN is parallel to OB

Answer: (a) $\mathbf{\frac{3}{4}a + \frac{1}{4}b}$ **(b) MN =** $\mathbf{\frac{1}{4}b}$**, parallel to OB**

3 Changing the subject

Make r the subject of $P = 2\sqrt{r - 1} - 4$

Add 4: $P + 4 = 2\sqrt{r - 1}$

Divide by 2: $\frac{P + 4}{2} = \sqrt{r - 1}$

Square both sides: $\frac{(P + 4)^2}{4} = r - 1$

$r = \frac{(P + 4)^2}{4} + 1$

Answer: $\mathbf{r = \frac{(P + 4)^2}{4} + 1}$

4 Matrix of a transformation

Write down the matrix that represents each transformation.
(a) Rotation of 90° clockwise about the origin
(b) Reflection in the line y = x

(a) Find where (1, 0) and (0, 1) go: they are the columns of the matrix

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Answer: (a) $\mathbf{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}$ **(b)** $\mathbf{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$

5 Speed and a moving train

A train travels at a constant speed of 108 km/h.
It takes 55 seconds to cross completely over a bridge which is 1,250 m long.
Work out the length of the train, in metres.

Speed = 108 km/h = 108,000 ÷ 3600 = 30 m/s
Distance travelled in 55 s = 30 × 55 = 1,650 m
This is the length of the bridge plus the length of the train
Train = 1,650 − 1,250 = 400 m
Answer: 400 m

6 Stationary points of y = ax + b/x

$y = 4x + \frac{4}{x}$

Find the coordinates of the stationary points and determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = 4 - \frac{4}{x^2}$

$\frac{dy}{dx} = 0$: $\frac{4}{x^2} = 4$, so $x^2 = 1$ and $x = 1$ or $x = -1$

$y = 8$ when $x = 1$ and $y = -8$ when $x = -1$

$\frac{d^2y}{dx^2} = \frac{8}{x^3}$

$x = 1$: $\frac{d^2y}{dx^2} = 8$ so minimum; $x = -1$: $\frac{d^2y}{dx^2} = -8$ so maximum

Answer: (1, 8) minimum; (−1, −8) maximum

7 Simultaneous equations with an inverse matrix

$M = \begin{pmatrix} 2 & -2 \\ 1 & 2 \end{pmatrix}$.

(a) Find M^{-1}.
(b) Use your answer to (a) to solve
$2x - 2y = -4$
$x + 2y = -2$

(a) det M = 6 so $M^{-1} = \frac{1}{6}\begin{pmatrix} 2 & 2 \\ -1 & 2 \end{pmatrix}$

(b) The equations are $\begin{pmatrix} 2 & -2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{6}\begin{pmatrix} 2 & 2 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} -4 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

Answer: (a) $\frac{1}{6}\begin{pmatrix} 2 & 2 \\ -1 & 2 \end{pmatrix}$ **(b) x = −2, y = 0**

8 Proof with indices

Show that $7^{m+2} + 7^m$ is always a multiple of 50.

$7^{m+2} = 7^m \times 7^2$
$7^{m+2} + 7^m = 7^m(7^2 + 1)$
$7^2 + 1 = 50$
$= 50 \times 7^m$, which is a multiple of 50
Answer: 50×7^m

9 Wire cut in two

A piece of wire 28 cm long is cut into two pieces. One piece is bent to form a square of side x cm. The other piece is bent to form a rectangle whose length is three times its width.

(a) Show that the total area, A cm^2, enclosed by the two shapes is $A = x^2 + \frac{3(28 - 4x)^2}{64}$.

(b) Find the value of x for which A is least, and work out this least area.

Rectangle perimeter $28 - 4x = 8w$, so $w = \frac{28 - 4x}{8}$ and its area is $3w^2 = \frac{3(28 - 4x)^2}{64}$

$A = x^2 + \frac{3(28 - 4x)^2}{64} = \frac{7}{4}x^2 - \frac{21}{2}x + \frac{147}{4}$

$\frac{dA}{dx} = \frac{7}{2}x - \frac{21}{2} = 0$, so $x = 3$. $\frac{d^2A}{dx^2} = \frac{7}{2} > 0$, so a minimum

Square: 9, rectangle: $6 \times 2 = 12$, so $A = 21$

Answer: x = 3, least area 21 cm^2

10 Combining means

The mean of the heights in cm for the boys is 75.
The mean of the heights in cm for the girls is 72.
The mean for all of the boys and girls together is 74.4.
Work out the ratio of the number of boys to the number of girls.

Let there be p boys and q girls
Total = $75p + 72q$, and the overall mean gives $75p + 72q = 74.4(p + q)$
$0.6p = 2.4q$
$p : q = 2.4 : 0.6 = 4 : 1$

Answer: 4 : 1