Day 333 **IGCSE 10-a-day: Model Solutions**

1 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where $s = -t^3 - 8t^2 + 12t$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when t = 3.

$v = \frac{ds}{dt} = -3t^2 - 16t + 12$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = -6t - 16$

When t = 3: $v = -63$ and $a = -34$

Answer: (a) -63 m/s (b) -34 m/s^2

2 Mean of a changing set

The mean of 7 numbers is 115.

One number is removed and the mean of the remaining 6 numbers is 126.5.

Work out the number that was removed.

Total of the 7 numbers = $7 \times 115 = 805$

Total of the 6 remaining = $6 \times 126.5 = 759$

Number removed = $805 - 759 = 46$

Answer: 46

3 Solving quadratics by factorising

Solve $12x^2 - 6 = x$

Rearrange so that one side is 0: $12x^2 - x - 6 = 0$

Factorise: $(3x + 2)(4x - 3) = 0$

$3x + 2 = 0$ or $4x - 3 = 0$

$x = -\frac{2}{3}$ or $x = \frac{3}{4}$

Answer: $x = -\frac{2}{3}$ or $x = \frac{3}{4}$

4 Composite with an inverse function

$f(x) = 3x - 2$ and $g(x) = 3x^2 - 3x + 6$.

(a) Find $f^{-1}g(x)$.

(b) Work out $f^{-1}g(-3)$.

(a) $f^{-1}(x) = \frac{x + 2}{3}$

$f^{-1}g(x) = \frac{3x^2 - 3x + 6 + 2}{3} = \frac{3x^2 - 3x + 8}{3}$

(b) $g(-3) = 42$, so $f^{-1}g(-3) = \frac{42 + 2}{3}$

Answer: (a) $\frac{3x^2 - 3x + 8}{3}$ (b) $\frac{44}{3}$

5 Normal in the form ax + by + c = 0

The point A lies on the curve $y = 3x^2 - 5x - 4$ where x = 0.

Find the equation of the normal at A. Give your answer in the form ax + by + c = 0 where a, b and c are integers.

$y = -4$ at $x = 0$ and gradient of curve $= -5$

Gradient of normal $= \frac{1}{5}$

$y + 4 = \frac{1}{5}x$

$y = \frac{1}{5}x - 4$

Multiply by 5: $x - 5y - 20 = 0$

Answer: $x - 5y - 20 = 0$

6 Exact lengths and areas

A right-angled triangle has base $(7 + \sqrt{7})$ cm and height $(7 - \sqrt{7})$ cm.
Show that the area of the triangle is a rational number.
Work out the area.

Area $= \frac{1}{2} \times (7 + \sqrt{7})(7 - \sqrt{7})$

$(7 + \sqrt{7})(7 - \sqrt{7}) = 49 - 7 = 42$

Area $= \frac{1}{2} \times 42 = 21$

Answer: 21 cm^2

7 Exact values from a given ratio

x is an acute angle and $\sin x = \frac{5}{8}$.

Find the exact value of (a) cos x (b) tan x.
Give each answer as a fraction, rationalising any denominator.

Right-angled triangle: opposite = 5, hypotenuse = 8

Adjacent $= \sqrt{8^2 - 5^2} = \sqrt{39}$

(a) $\cos x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{39}}{8}$

(b) $\tan x = \frac{5}{\sqrt{39}} = \frac{5\sqrt{39}}{39}$

Answer: (a) $\frac{\sqrt{39}}{8}$ (b) $\frac{5\sqrt{39}}{39}$

8 Cone made from a sector

A sector of a circle has radius 12 cm and angle 180°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector $= \frac{180}{360} \times 2\pi \times 12 = 12\pi$ cm

This is the circumference of the base: $2\pi r = 12\pi$, so r = 6 cm

(b) Slant height = 12 cm, so height$^2 = 12^2 - 6^2 = 108$, height $= 6\sqrt{3}$ cm

Volume $= \frac{1}{3} \times \pi \times 6^2 \times 6\sqrt{3} = 72\sqrt{3}\pi$ cm^3

Answer: (a) 6 cm (b) $72\sqrt{3}\pi$ cm^3

9 Ratio and mixtures

Alloy A holds copper and tin in the ratio 2 : 1.
Alloy B holds copper and tin in the ratio 3 : 2.
24 kg from alloy A is mixed with some of alloy B.
The ratio of copper to tin in the mixture is 17 : 10.
Work out how many kg were taken from alloy B.

Let y kg be taken from B

copper: $16 + \frac{3y}{5}$

tin: $8 + \frac{2y}{5}$

Ratio 17 : 10, so $10(16 + \frac{3y}{5}) = 17(8 + \frac{2y}{5})$

$(\frac{-4}{5})y = -24$, so y = 30

Answer: 30 kg

10 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by

$h = -3t^2 + 24t + 5$

(a) Work out the greatest height reached by the ball.

(b) Work out the velocity of the ball when t = 7. State whether it is moving up or down.

$\frac{dh}{dt} = -6t + 24 = 0$ gives $t = 4$

$\frac{d^2h}{dt^2} = -6 < 0$ so maximum. $h = 53$

When $t = 7$: $\frac{dh}{dt} = 24 - 6 \times 7 = -18$

Answer: (a) 53 m (b) −18 m/s, moving down