Day 332 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $(p - q)^2 - (p + q)^2$
when $p = 5$, $q = -2$ and $r = -3$.

Substitute the values: $(5 - (-2))^2 - (5 + (-2))^2$
$= 49 - 9$
$= 40$
Answer: 40

2 Arc length and sector area

A sector of a circle has radius 4 cm and angle 200°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{200}{360}$

Arc length = $\frac{200}{360} \times 2 \times \pi \times 4 = \frac{40}{9}\pi$ cm

(b) Area = $\frac{200}{360} \times \pi \times 4^2 = \frac{80}{9}\pi$ cm^2

Answer: (a) $\frac{40}{9}\pi$ cm (b) $\frac{80}{9}\pi$ cm^2

3 Error intervals

The width of a table is 12 cm, truncated to a whole number.
Write down the error interval for the width of a table, using w.

Truncating cuts off the later digits without rounding.
The smallest possible value is 12 and the value must be less than 13
$12 \leq w < 13$
Answer: $12 \leq w < 13$

4 Tangents parallel to a line

A curve has equation $y = x^3 - 12x^2 + 31x + 8$.
Find the equations of the two tangents to the curve that are parallel to the line $y = -5x - 4$.

Parallel lines have equal gradients: gradient = -5
$\frac{dy}{dx} = 3x^2 - 24x + 31 = -5$, so $3x^2 - 24x + 36 = 0$
$3(x - 2)(x - 6) = 0$ gives $x = 2$ or $x = 6$
At $x = 2$: $y = 30$, tangent $y - 30 = -5(x - 2)$
At $x = 6$: $y = -22$, tangent $y - (-22) = -5(x - 6)$
Answer: $y = -5x + 40$ and $y = -5x + 8$

5 Compound interest: how many years

£1,000 is invested at 3% per year compound interest.
Work out the least whole number of years after which the investment is worth more than £1,250. (Calculator)

Value after n years = 1000×1.03^n
After 7 years: £1,229.87 (less than £1,250)
After 8 years: £1,266.77 (more than £1,250)
Answer: 8 years

6 Compound interest

Kira invests £8,000. The rates of interest for the next 3 years are 3%, 5% and 6%.
Compound interest is paid. Work out the value of the investment after 3 years.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.03, 1.05 and 1.06
$8000 \times 1.03 \times 1.05 \times 1.06 = 9171.12$
To the nearest penny: £9,171.12
Answer: £9,171.12

7 Find constants from a turning point

The curve $y = x^3 + ax^2 + bx + 4$ has stationary points at $x = 2$ and $x = 4$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$

$x = 2$: $12 + 4a + b = 0$. $x = 4$: $48 + 8a + b = 0$

Subtract the equations: $-36 + -4a = 0$, so $a = -9$

Then $b = 24$

Answer: $a = -9$, $b = 24$

8 Composite functions: fg and gf

$f(x) = -x - 1$ and $g(x) = x^2 - 1$
Solve $fg(x) = gf(x)$

$fg(x) = f(x^2 - 1) = -x^2$

$gf(x) = g(-x - 1) = (-x - 1)^2 - 1 = x^2 + 2x$

Set them equal and rearrange: $2x^2 + 2x = 0$

Divide by 2 and factorise: $x(x + 1) = 0$

$x = -1$ or $x = 0$

Answer: $x = -1$ or $x = 0$

9 Three stationary points of a quartic

$y = x^4 - 2x^2 - 6$
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 4x^3 - 4x = 4x(x - 1)(x + 1)$

$\frac{dy}{dx} = 0$ gives $x = -1$, $x = 0$ or $x = 1$

Points: $(-1, -7)$, $(0, -6)$, $(1, -7)$

$\frac{d^2y}{dx^2} = 12x^2 - 4$

$x = -1$: $\frac{d^2y}{dx^2} = 8$; $x = 0$: $\frac{d^2y}{dx^2} = -4$; $x = 1$: $\frac{d^2y}{dx^2} = 8$

So $(-1, -7)$ is a minimum, $(0, -6)$ is a maximum, $(1, -7)$ is a minimum

Answer: $(-1, -7)$ minimum; $(0, -6)$ maximum; $(1, -7)$ minimum

10 Bounds of a compound expression

$p = 10$ and $r = 3.6$, each correct to 1 decimal place.
$q = 2.65$, correct to 2 decimal places.

Work out the lower bound of $\frac{p - q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the lower bound use LB of the top and UB of the bottom

Top: LB p − UB $q = 9.95 - 2.655 = 7.295$

Bottom: UB $r = 3.65$

$7.295 \div 3.65 = 2.00$

Answer: 2.00