Day 331 **IGCSE 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

83%, $\frac{12}{13}$, $\frac{1}{3}$, $\frac{6}{7}$

Convert each to a decimal: 83% = 0.83, $\frac{12}{13}$ = 0.923, $\frac{1}{3}$ = 0.333, $\frac{6}{7}$ = 0.857

In order: $\frac{1}{3}$, 83%, $\frac{6}{7}$, $\frac{12}{13}$

Answer: $\frac{1}{3}$, 83%, $\frac{6}{7}$, $\frac{12}{13}$

2 Gradient with fractional or negative powers

The equation of a curve is $y = 2x^2 - 5x + \frac{4}{x}$.

Work out the gradient of the curve at the point where $x = -4$.

$\frac{dy}{dx} = 4x - 5 - 4x^{-2}$

When $x = -4$: gradient $= -\frac{85}{4}$

Answer: $-\frac{85}{4}$

3 Percentage change

The price of a phone is increased by 40%.
The new price is then decreased by 5%.
Work out the overall percentage change in the price.

Multipliers: 1.4 and 0.95
Overall multiplier = 1.4 × 0.95 = 1.33
1.33 − 1 = 0.33, so an overall increase of 33%
Answer: 33% increase

4 Rate of cooling or growth

The temperature, T °C, of a cup of tea t minutes after it is removed from a heat source is modelled by

$T = 25 + \frac{64}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when t = 4.

(b) Find the value of t when the temperature is falling at 64 °C per minute.

$T = 25 + 64t^{-1}$, so $\frac{dT}{dt} = -64t^{-2}$

When t = 4: $\frac{dT}{dt} = -4$

$-\frac{64}{t^2} = -64$, so $t^2 = 1$ and t = 1

Answer: (a) −4 °C/min (b) t = 1

5 Area of a triangle using sine

In triangle PQR, PQ = 11 cm and angle PQR = 135°.
The area of the triangle is $11\sqrt{2}$ cm^2.
Work out the length of QR.
Do not use a calculator.

Area = $\frac{1}{2} \times PQ \times QR \times \sin Q$

$11\sqrt{2} = \frac{1}{2} \times 11 \times QR \times \sin 135°$

$\sin 135° = \frac{\sqrt{2}}{2}$, so $11\sqrt{2} = \frac{11\sqrt{2}}{4} \times QR$

$QR = 11\sqrt{2} \div \frac{11\sqrt{2}}{4} = 4$ cm

Answer: 4 cm

6 Line and curve intersections

Solve the simultaneous equations
$y = x^2 + 2x + 4$
$y = -5x - 8$

Substitute: $x^2 + 2x + 4 = -5x - 8$
Rearrange: $x^2 + 7x + 12 = 0$
Factorise: $(x + 3)(x + 4) = 0$, so $x = -3$ or $x = -4$
When $x = -3$: $y = -5 \times (-3) - 8 = 7$
When $x = -4$: $y = -5 \times (-4) - 8 = 12$
Answer: $x = -4$, $y = 12$ and $x = -3$, $y = 7$

7 Tree diagrams and reverse conditional probability

The probability that it rains is $\frac{3}{10}$.

If it rains, the probability that Sam is late is $\frac{1}{6}$. If it does not rain, the probability is $\frac{1}{2}$.

(a) Work out the probability that Sam is late.
(b) Given that Sam is late, work out the probability that it rained.

(a) P(rain and late) = $\frac{3}{10} \times \frac{1}{6} = \frac{1}{20}$; P(no rain and late) = $\frac{7}{10} \times \frac{1}{2} = \frac{7}{20}$

P(late) = $\frac{1}{20} + \frac{7}{20} = \frac{2}{5}$

(b) P(rain | late) = $\frac{\frac{1}{20}}{\frac{2}{5}} = \frac{1}{8}$

Answer: (a) $\frac{2}{5}$ (b) $\frac{1}{8}$

8 Increasing for all x

$f(x) = x^3 + kx^2 + 48x - 4$, where k is a constant.
Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 48$
f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots
Discriminant: $(2k)^2 - 4 \times 3 \times 48 < 0$, so $4k^2 < 576$
$k^2 < 144$, so $-12 < k < 12$
Answer: $-12 < k < 12$

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9 A squared in terms of A and I

$A = \begin{pmatrix} 1 & -4 \\ 1 & 5 \end{pmatrix}$.

$A^2 = pA + qI$, where I is the 2 × 2 identity matrix.
Find the value of p and the value of q.

$A^2 = \begin{pmatrix} -3 & -24 \\ 6 & 21 \end{pmatrix}$

Top right entries: $-24 = p \times (-4)$, so $p = 6$
Top left entries: $-3 = 6 \times 1 + q$, so $q = -9$
Check bottom right: $6 \times 5 - 9 = 21$
Answer: p = 6, q = −9

10 Average speed

Leila drives 75 km at an average speed of 25 km/h.
The drive then continues for another 40 km at an average speed of 20 km/h.
Work out the average speed for the whole journey.

Time for first part = 75 ÷ 25 = 3 hours
Time for second part = 40 ÷ 20 = 2 hours
Total distance = 115 km, total time = 5 hours
Average speed = 115 ÷ 5 = 23 km/h
Answer: 23 km/h