Day 330 **IGCSE 10-a-day: Model Solutions**

1 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where

$s = -t^2 + 7t + 9 + \frac{12}{t}$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when t = 1.

$v = \frac{ds}{dt} = -2t + 7 - 12t^{-2}$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = -2 + 24t^{-3}$

When t = 1: v = −7 and a = 22

Answer: (a) −7 m/s (b) 22 m/s^2

2 Median and quartiles from a list

Here are 11 numbers.

49, 7, 32, 45, 3, 42, 37, 56, 12, 12, 11

Find (a) the median (b) the interquartile range.

Ordered: 3, 7, 11, 12, 12, 32, 37, 42, 45, 49, 56

(a) Median is the 6th value = 32

(b) Lower quartile is the 3rd value = 11; upper quartile is the 9th value = 45

IQR = 45 − 11 = 34

Answer: (a) 32 (b) 34

3 Point with a given gradient (quadratic)

A curve has equation $y = -2x^2 - 3x - 4$.

The gradient at the point P on the curve is −19.

Find the coordinates of P.

$\frac{dy}{dx} = -4x - 3$

$-4x - 3 = -19$, so $-4x = -16$

x = 4

y = −48

Answer: (4, −48)

4 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)

(a) Work out angle ACB, marked θ, to 1 decimal place.

(b) Work out the area of triangle ABC to 3 significant figures.

The diagram is not drawn accurately.

(a) $\cos\theta = \frac{12^2 + 16^2 - 10^2}{2 \times 12 \times 16} = \frac{300}{384}$

θ = 38.6° (1 d.p.)

(b) Area $= \frac{1}{2} \times 12 \times 16 \times \sin 38.6°$

= 59.9 cm^2 (3 s.f.)

Answer: (a) 38.6° (b) 59.9 cm^2

5 Product of brackets: tangent gradient

y = x(x − 1)(x + 1)

Work out the gradient of the curve at the point where x = −1.

Expand: $y = x^3 - x$

$\frac{dy}{dx} = 3x^2 - 1$

x = −1: $\frac{dy}{dx} = 2$

Answer: 2

6 Solve f inverse equals g

$f(x) = \frac{x + 3}{3}$ and $g(x) = x^2 + 6x - 1$.

Solve $f^{-1}(x) = g(x)$.

$f^{-1}(x) = 3x - 3$

$3x - 3 = x^2 + 6x - 1$

$x^2 + 3x + 2 = 0$

$(x + 1)(x + 2) = 0$

$x = -2$ or $x = -1$

Answer: $x = -2$ or $x = -1$

7 Density of an alloy

An alloy is made by mixing 100 cm^3 of aluminium with 100 cm^3 of tin.
The density of aluminium is 2.7 g/cm^3. The density of tin is 7.3 g/cm^3.
Work out the density of the alloy.

Mass of aluminium = $100 \times 2.7 = 270$ g

Mass of tin = $100 \times 7.3 = 730$ g

Total mass = 1000 g, total volume = 200 cm^3

Density = $1000 \div 200 = 5$ g/cm^3

Answer: 5 g/cm^3

8 Tangent to a circle: proof

Show that the line $y = 3x - 30$ is a tangent to the circle $x^2 + y^2 = 90$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = 3x - 30$ into $x^2 + y^2 = 90$: $x^2 + (3x - 30)^2 = 90$

Expand: $10x^2 - 180x + 810 = 0$

$b^2 - 4ac = (-180)^2 - 4 \times 10 \times 810 = 32400 - 32400 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-(-180)}{2 \times 10} = 9$, and $y = 3 \times 9 - 30 = -3$. The point is $(9, -3)$

Answer: $(9, -3)$

9 Angles of elevation

A point A is on level ground in line with the base of a cliff. (Calculator)
The angle of elevation of the top of the cliff from A is 45°.
B is a point on the same line, 80 m further from the cliff than A.
The angle of elevation of the top of the cliff from B is 17°.
Work out the height of the cliff. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 45° = \frac{h}{x}$, so $x = h \div \tan 45°$

From B: $\tan 17° = \frac{h}{x + 80}$, so $x + 80 = h \div \tan 17°$

Subtract: $80 = h \div \tan 17° - h \div \tan 45° = h(3.2709 - 1.0000)$

$h = 80 \div 2.2709 = 35.2$ m (1 d.p.)

Answer: 35.2 m

10 Counting factors

$N = 2^2 \times 3 \times 5$
How many factors does N have?
Include 1 and N.

Add 1 to each power and multiply

$(2 + 1) \times (1 + 1) \times (1 + 1) = 3 \times 2 \times 2 = 12$

Answer: 12