Day 33 **IGCSE 10-a-day: Model Solutions**

1 Stationary points of a quartic

A curve has equation $y = x^4 - 18x^2 - 3$.
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 4x^3 - 36x = 4x(x - 3)(x + 3)$

$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 3$ or $x = -3$

$y = -3$ when $x = 0$, and $y = -84$ when $x = \pm 3$

$\frac{d^2y}{dx^2} = 12x^2 - 36$: it is -36 at $x = 0$ (maximum) and 72 at $x = \pm 3$ (minimum)

Answer: (0, −3) maximum; (3, −84) and (−3, −84) minima

2 Adding and scaling matrices

$A = \begin{pmatrix} 6 & -5 \\ 5 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & -5 \\ -4 & 0 \end{pmatrix}$.
Find the matrix X such that A + X = B.

$X = B - A$

$X = \begin{pmatrix} -2 & -5 \\ -4 & 0 \end{pmatrix} - \begin{pmatrix} 6 & -5 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} -8 & 0 \\ -9 & -4 \end{pmatrix}$

Answer: $\mathbf{X} = \begin{pmatrix} \mathbf{-8} & \mathbf{0} \\ \mathbf{-9} & \mathbf{-4} \end{pmatrix}$

3 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 5^2 + 20^2 = 425$
$AG^2 = AC^2 + CG^2 = 425 + 4^2 = 441$
AG = 21 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{4}{\sqrt{425}}$

angle GAC = 11.0° (1 d.p.)
Answer: (a) 21 cm (b) 11.0°

4 Averages with algebra

Here are five numbers: 2x + 4, 3x − 2, x + 4, 2x + 1, x + 2.
The mean of the five numbers is 9.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = (2x + 4) + (3x − 2) + (x + 4) + (2x + 1) + (x + 2) = 9 × 5 = 45
9x + 9 = 45, so 9x = 36 and x = 4
The numbers are 12, 10, 8, 9, 6
(b) In order: 6, 8, 9, 10, 12, so the median is 9
Answer: (a) x = 4 (b) 9

5 Increasing and decreasing intervals

$y = -2x^3 + 6x - 1$

Find the range of values of x for which y is increasing.

$\frac{dy}{dx} = -6x^2 + 6 = -6(x + 1)(x - 1)$

$\frac{dy}{dx} = 0$ when $x = -1$ and $x = 1$

The graph of $\frac{dy}{dx}$ against x is a ∩-shaped quadratic

y is increasing when $\frac{dy}{dx} > 0$ and decreasing when $\frac{dy}{dx} < 0$

So y is increasing when $-1 < x < 1$

Answer: $-1 < x < 1$

6 Vector of given length parallel to another

$\mathbf{p} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

Find a vector of length 20 that is parallel to $\mathbf{p}$.

$|\mathbf{p}| = \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5$

Scale factor = 20 ÷ 5 = 4

$4\mathbf{p} = \begin{pmatrix} 16 \\ 12 \end{pmatrix}$

Answer: $\begin{pmatrix} 16 \\ 12 \end{pmatrix}$ (or $\begin{pmatrix} -16 \\ -12 \end{pmatrix}$)

7 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.

OP = $-2\mathbf{a} - 2\mathbf{b}$, OQ = $-11\mathbf{a} + 10\mathbf{b}$ and OR = $-17\mathbf{a} + 18\mathbf{b}$.

(a) Show that P, Q and R lie on a straight line.

(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-9\mathbf{a} + 12\mathbf{b}$

QR = OR − OQ = $-6\mathbf{a} + 8\mathbf{b}$

QR = $\frac{2}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line

(b) PQ : QR = 3 : 2

Answer: (b) 3 : 2

8 Increasing range for y = ax + b/x

$y = x + \frac{1}{x}$

Find the values of x for which y is increasing.

$\frac{dy}{dx} = 1 - \frac{1}{x^2}$

$\frac{dy}{dx} > 0$: $1 > \frac{1}{x^2}$

$x^2 > 1$ (x^2 is positive, so multiply through)

$x < -1$ or $x > 1$

Answer: $x < -1$ or $x > 1$

9 Counting factors

$N = 2 \times 3^5 \times 5^2$

How many of the factors of N are even numbers?

An even factor must contain at least one 2
Power of 2: 1 to 1, that is 1 choices
Power of 3: 0 to 5, that is 6 choices
Power of 5: 0 to 2, that is 3 choices
Total = 1 × 6 × 3 = 18
Answer: 18

10 Trigonometric equations

Solve $5\cos x° - 1 = -1$ for $0 \leq x \leq 360$

Rearrange: 5 cos x° = 0, so cos x° = 0
From the graph of y = cos x°, cos x° = 0 when x = 90 or 270
Answer: x = 90, 270