Day 329 **IGCSE 10-a-day: Model Solutions**

1 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle OAB = 59°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) OA = OB (radii), so triangle OAB is isosceles and angle OBA = 59°
Angle AOB = $180 - 2 \times 59 = 62°$
(b) The angle at the circumference is half the angle at the centre: $62 \div 2 = 31°$
Answer: (a) 62° (b) 31°

2 Vector between two points

The position vectors of A and B are $\begin{pmatrix} -4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 8 \end{pmatrix}$.

(a) Write down the vector **AB**.
(b) Find |**AB**|, giving the answer as a surd in simplest form if it is not exact.

(a) $\mathbf{AB} = \begin{pmatrix} -1 \\ 8 \end{pmatrix} - \begin{pmatrix} -4 \\ -5 \end{pmatrix} = \begin{pmatrix} 3 \\ 13 \end{pmatrix}$
(b) $|\mathbf{AB}| = \sqrt{(3)^2 + (13)^2} = \sqrt{178}$
Answer: (a) $\begin{pmatrix} \mathbf{3} \\ \mathbf{13} \end{pmatrix}$ **(b)** $\sqrt{\mathbf{178}}$

3 Quadratic graphs

The curve $y = 2x^2 - 24x + 40$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the minimum point of the curve.

(a) Put y = 0. $2x^2 - 24x + 40 = 0$, so $2(x^2 + 12x + 20) = 0$, giving $2(x - 2)(x - 10) = 0$
$x - 2 = 0$ or $x - 10 = 0$, so $x = 2$ or $x = 10$
(b) The turning point is halfway between A and B: $x = (2 + 10) \div 2 = 6$
When x = 6: $y = 2 \times 6^2 - 24 \times 6 + 40 = -32$
Answer: (a) A(2, 0), B(10, 0) (b) (6, −32)

4 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by
$h = -4t^2 + 8t$
(a) Work out the greatest height reached by the ball.
(b) Work out the velocity of the ball when t = 2. State whether it is moving up or down.

$\frac{dh}{dt} = -8t + 8 = 0$ gives t = 1
$\frac{d^2h}{dt^2} = -8 < 0$ so maximum. h = 4
When t = 2: $\frac{dh}{dt} = 8 - 8 \times 2 = -8$
Answer: (a) 4 m (b) −8 m/s, moving down

5 Exact lengths and areas

A square has sides of length $(5 + \sqrt{2})$ cm.
Work out the area of the square.
Give your answer in the form $a + b\sqrt{2}$

Area $= (5 + \sqrt{2})^2$
$= 25 + 10\sqrt{2} + 2$
$= 27 + 10\sqrt{2}$
Answer: $\mathbf{27 + 10\sqrt{2}\ cm^2}$

6 Rate of cooling or growth

The temperature, T °C, of a cup of tea t minutes after it is removed from a heat source is modelled by

$T = 25 + \frac{48}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when t = 4.

(b) Find the value of t when the temperature is falling at 12 °C per minute.

$T = 25 + 48t^{-1}$, so $\frac{dT}{dt} = -48t^{-2}$

When t = 4: $\frac{dT}{dt} = -3$

$-\frac{48}{t^2} = -12$, so $t^2 = 4$ and t = 2

Answer: (a) −3 °C/min (b) t = 2

7 Exponential graph from points

The diagram shows part of the graph of $y = ab^x$.
The curve passes through (0, 32) and (2, 2).
(a) Find the values of a and b. (b) Work out the value of y when x = 3.

(a) At x = 0: $y = a \times b^0 = a$, so a = 32
At x = 2: $32 \times b^2 = 2$ so $b^2 = 0.0625$
b = 0.25
(b) $y = 32 \times 0.25^3 = 0.5$
Answer: (a) a = 32, b = 0.25 (b) 0.5

8 Estimating the median from grouped data

The table gives information about 80 times.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 23, 36, 41, 64, 80
The median is the value at position 80 ÷ 2 = 40
It is in the class 30 to 35, with 36 values before it, so we need 4 more of the 5 values in this class
Median $\approx 30 + \frac{4}{5} \times 5 = 34$

Answer: 34

9 Same gradient on two curves

Two curves have equations $y = 2x^2 + 4x - 2$ and $y = x^2 + 12x + 7$.
Find the value of x at which the curves have the same gradient.

First curve: $\frac{dy}{dx} = 4x + 4$

Second curve: $\frac{dy}{dx} = 2x + 12$

Equal gradients: 4x + 4 = 2x + 12
2x = 8, so x = 4
Answer: x = 4

10 Functions that are their own inverse

$f(x) = \frac{3x - 5}{2x - 3}$ where $x \neq \frac{3}{2}$

(a) Show that ff(x) = x.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by $(2x - 3)$

$ff(x) = \frac{3(3x - 5) + (-5)(2x - 3)}{2(3x - 5) - 3(2x - 3)}$

Top $= 9x - 15 - 10x + 15 = -x$

Bottom $= 6x - 10 - 6x + 9 = -1$

$ff(x) = \frac{-x}{-1} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{3x - 5}{2x - 3}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{3x - 5}{2x - 3}}$