Day 328 **IGCSE 10-a-day: Model Solutions**

1 Distance-time graphs

The graph shows Fatima's cycle ride to a café and back.
(a) Work out Fatima's speed on the way back.
(b) Work out Fatima's average speed for the whole ride, including the stop.

(a) Return: 24 km in 1 hours, so speed = 24 ÷ 1 = 24 km/h
(b) Total distance = 2 × 24 = 48 km, total time = 3 hours
Average speed = 48 ÷ 3 = 16 km/h
Answer: (a) 24 km/h (b) 16 km/h

2 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where $s = 3t^3 + 5t^2 + 3t + 2$ $(t > 0)$
Find (a) the velocity of P, (b) the acceleration of P, when t = 3.

$v = \frac{ds}{dt} = 9t^2 + 10t + 3$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 18t + 10$

When t = 3: v = 114 and a = 64
Answer: (a) 114 m/s (b) 64 m/s^2

3 Expanding three brackets

Expand and simplify $(x + 4)(x - 3)(x - 2)$

Expand the first two brackets: $(x + 4)(x - 3) = x^2 + x - 12$
Multiply by $(x - 2)$: $x(x^2 + x - 12) - 2(x^2 + x - 12)$
$= x^3 - x^2 - 14x + 24$
Answer: $x^3 - x^2 - 14x + 24$

4 Direct variation

y is directly proportional to the cube of x.
When x = 8, y = 1536.
(a) Find a formula for y in terms of x.
(b) Find y when x = 3.

$y = kx^3$
$1536 = k \times 8^3$ so k = 3
$y = 3x^3$
$y = 3 \times 27 = 81$
Answer: (a) $y = 3x^3$ (b) y = 81

5 Angles of elevation

A point A is on level ground in line with the base of a tower. (Calculator)
The angle of elevation of the top of the tower from A is 49°.
B is a point on the same line, 50 m further from the tower than A.
The angle of elevation of the top of the tower from B is 26°.
Work out the height of the tower. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 49° = \frac{h}{x}$, so $x = h \div \tan 49°$

From B: $\tan 26° = \frac{h}{x + 50}$, so $x + 50 = h \div \tan 26°$

Subtract: $50 = h \div \tan 26° - h \div \tan 49° = h(2.0503 - 0.86929)$
h = 50 ÷ 1.1810 = 42.3 m (1 d.p.)
Answer: 42.3 m

6 Rationalising the denominator

Write $\frac{3+\sqrt{2}}{2-\sqrt{2}}$ in the form $p + q\sqrt{2}$
where p and q are rational numbers.

Multiply top and bottom by $(2 + \sqrt{2})$:
Numerator: $(3 + \sqrt{2})(2 + \sqrt{2}) = 8 + 5\sqrt{2}$
Denominator: $4 - 2 = 2$
$= \frac{8 + 5\sqrt{2}}{2}$

Answer: $\mathbf{\frac{8 + 5\sqrt{2}}{2}}$

7 Largest domain and range

$f(x) = \frac{1}{\sqrt{x-4}}$.
(a) State the largest possible domain of f.
(b) State the range of f for this domain.

(a) The square root must be positive (it is in the denominator): $x - 4 > 0$
Domain: $x > 4$
(b) 1 ÷ (a positive number) is positive, and can be as large as we like or as close to 0 as we like
Answer: (a) $x > 4$ (b) $f(x) > 0$

8 No stationary points

A curve has equation $y = x^3 + 9x^2 + 30x + 5$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 3x^2 + 18x + 30 = 3(x + 3)^2 + 3$

A squared term is never negative, so $\frac{dy}{dx} \geq 3 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 3 (when $x = -3$)

Answer: (b) 3

9 Bounds of speed, density and pressure

A force of 2400 N acts on a surface, correct to the nearest 50.
The area of the surface is 9.9 m^2, correct to 2 significant figures.
Work out the upper bound of the pressure.
Give your answer correct to 3 significant figures. (Calculator)

$2375 \leq$ force < 2425; $9.85 \leq$ area < 9.95
Upper bound of pressure = biggest force ÷ smallest area = 2425 ÷ 9.85
= 246.193... = 246 (3 s.f.)

Answer: 246 N/m^2

10 Stationary point of y = ax + b/√x

A curve has equation $y = 2x + \frac{4}{\sqrt{x}}$ for $x > 0$.

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 2 - 2x^{-3/2}$

$\frac{dy}{dx} = 0$: $2x^{-3/2} = 2$, so $x^{3/2} = 1$

$x = 1^{2/3} = 1$, and $y = 6$

$\frac{d^2y}{dx^2} = 3x^{-5/2}$

At $x = 1$, $\frac{d^2y}{dx^2} = 3$, which is positive, so a minimum

Answer: (1, 6), minimum