Day 327 **IGCSE 10-a-day: Model Solutions**

1 Bearings with sine and cosine rules

A ship sails 15 km from P to Q on a bearing of 063°.
It then sails 10 km from Q to R on a bearing of 153°.
(a) Work out PR (3 s.f.). (b) Work out the bearing of R from P (nearest degree). (Calculator)

Angle PQR = 063° + 180° − 153° = 90°

$PR^2 = 15^2 + 10^2 - 2 \times 15 \times 10 \times \cos 90°$ so PR = 18.0 km

$\cos QPR = \frac{15^2 + 18.028^2 - 10^2}{2 \times 15 \times 18.028}$, angle QPR = 33.69°

Bearing of R from P = 063° + 33.69° = 097°

Answer: (a) 18.0 km (b) 097°

2 Increasing or decreasing at a point

$y = -x^3 - 6x^2 - 9x + 3$
At the point where $x = -2$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = -3x^2 - 12x - 9$

When $x = -2$: $\frac{dy}{dx} = 3$

3 is positive, so the curve is increasing

Answer: increasing (gradient 3)

3 Laws of indices

Given that $\frac{25 \times 5^4}{25^2} = 5^n$,

find the value of n.

$25 = 5^2$

Numerator: $5^2 \times 5^4 = 5^6$

Denominator: 5^4

Subtract the powers: 6 − 4 = 2

Answer: n = 2

4 Coordinates and gradient at the y-intercept

$y = 2x^4 - 5x^3 - x^2 + 7$
The curve crosses the y-axis at the point A.
Work out the coordinates of A and the gradient of the curve at A.

When x = 0, y = 7, so A is (0, 7)

$\frac{dy}{dx} = 8x^3 - 15x^2 - 2x$

When x = 0, $\frac{dy}{dx} = 0$

Answer: A = (0, 7), gradient 0

5 Variation in context

The braking distance d metres of a car is directly proportional to the square of its speed v km/h.
When v = 7, d = 73.5m.
Find the value of d when v = 3.

$d = kv^2$

$73.5 = k \times 7^2$ so $k = \frac{3}{2}$

$d = \frac{3}{2}v^2$

$d = \frac{3}{2} \times 9 = 13.5$

Answer: 13.5m

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6 Bounds in geometry

A runner covers 600 metres, correct to the nearest 10 metres, in a time of 11.1 seconds, correct to 1 decimal place. (Calculator)
Work out the lower bound of the runner's average speed in metres per second. Give your answer correct to 3 significant figures.

Distance is between 595 and 605 metres; time is between 11.05 and 11.15 seconds
Speed = distance ÷ time. For the lower bound use the smallest distance and the greatest time
595 ÷ 11.15 = 53.4 m/s (3 s.f.)
Answer: 53.4 m/s

7 Line and curve intersections

Solve the simultaneous equations
$y = x^2 + 2x - 3$
$y = x - 1$

Substitute: $x^2 + 2x - 3 = x - 1$
Rearrange: $x^2 + x - 2 = 0$
Factorise: $(x - 1)(x + 2) = 0$, so $x = 1$ or $x = -2$
When $x = 1$: $y = 1 \times 1 - 1 = 0$
When $x = -2$: $y = 1 \times (-2) - 1 = -3$
Answer: $x = -2$, $y = -3$ and $x = 1$, $y = 0$

8 Minimum distance with 1/t

A particle P moves on a straight line. At time t seconds ($t > 0$), its distance from a point O is s metres, where
$s = 2t^2 + \frac{32}{t}$
Show that P is at its minimum distance from O when $t = 2$, and find this minimum distance.

$s = 2t^2 + 32t^{-1}$, so $v = \frac{ds}{dt} = 4t - 32t^{-2}$

$v = 0$: $4t = \frac{32}{t^2}$, so $t^3 = 8$ and $t = 2$

$\frac{d^2s}{dt^2} = 4 + \frac{64}{t^3}$, which is 12 when $t = 2$, so a minimum
$s = 24$
Answer: 24 m

9 Bounds of roots

The area of a circle is 272 cm^2, correct to the nearest cm^2.
Work out the lower bound of the radius of the circle.
Give your answer correct to 3 significant figures. (Calculator)

$271.5 \leq \text{area} < 272.5$

Area = πr^2, so $r = \sqrt{\frac{\text{area}}{\pi}}$

Lower bound = $\sqrt{\frac{271.5}{\pi}} = 9.29630... = 9.30$ (3 s.f.)

Answer: 9.30 cm

10 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 22.
Work out the value of x.

Mid-points: 12.5, 17.5, 22.5, 27.5, 32.5
Estimated mean = $\Sigma fm \div \Sigma f$, so $(605 + 27.5x) \div (30 + x) = 22$
$605 + 27.5x = 22(30 + x) = 660 + 22x$
$(27.5 - 22)x = 660 - 605$
$5.5x = 55$, so $x = 10$
Answer: x = 10