Day 326 **IGCSE 10-a-day: Model Solutions**

1 Fencing against a wall

A farmer uses 24 m of fencing to enclose a rectangle against a wall, as shown.

(a) Show that the area, A m^2, is $A = 24x - 2x^2$.

(b) Find the maximum area, proving it is a maximum.

Side parallel to the wall = $24 - 2x$, so $A = x(24 - 2x) = -2x^2 + 24x$

$\frac{dA}{dx} = -4x + 24 = 0$ gives $x = 6$

$\frac{d^2A}{dx^2} = -4 < 0$, so a maximum

$A = 24 \times 6 - 2 \times 6^2 = 72$

Answer: 72 m^2

2 Solving trigonometric equations

Solve $2 \cos x = 0.4$ for $0 \leq x \leq 360$.

Give your answers correct to 1 decimal place. (Calculator)

$\cos x = 0.4 \div 2 = 0.2$

Principal value: $x = \cos^{-1}(0.2) = 78.5°$

Second solution: $x = 360° -$ (principal value)

All solutions in the range: 78.5, 281.5

Answer: x = 78.5, 281.5

3 Capture and recapture

20 fish in a lake are caught, tagged and put back.

Later, a second sample of 120 fish is caught. 3 of these are tagged.

(a) Work out an estimate for the number of fish in a lake.

(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 3 out of 120

This should match the proportion tagged in the whole population: $\frac{3}{120} = \frac{20}{N}$

$N = 20 \times 120 \div 3 = 800$

(b) No fish are born, die or leave between the samples, and the fish mix evenly

Answer: (a) 800 (b) The population stays the same between samples

4 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)

(a) Work out the length of the diagonal AG.

(b) Work out the angle between AG and the base ABCD, to 1 decimal place.

The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 8^2 + 12^2 = 208$

$AG^2 = AC^2 + CG^2 = 208 + 9^2 = 289$

$AG = 17$ cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{9}{\sqrt{208}}$

angle GAC = 32.0° (1 d.p.)

Answer: (a) 17 cm (b) 32.0°

5 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are $(x - 2)$ cm, $(x - 3)$ cm and $(x - 1)$ cm.
The longest side is $(x - 1)$ cm.
(a) Show that $x^2 - 8x + 12 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x - 2)^2 + (x - 3)^2 = (x - 1)^2$
Expand: $x^2 - 4x + 4 + x^2 - 6x + 9 = x^2 - 2x + 1$
Rearrange: $x^2 - 8x + 12 = 0$
(b) Factorise: $(x - 6)(x - 2) = 0$, so $x = 6$ (the other root, $x = 2$, does not give valid side lengths)
The shorter sides are 4 cm and 3 cm, so area $= \frac{1}{2} \times 4 \times 3 = 6$ cm^2

Answer: (b) 6 cm^2

6 Proof with indices

Show that $3^{m+3} - 3^m$ is a multiple of 26 for all integer values of m.

$3^{m+3} = 3^m \times 3^3$
$3^{m+3} - 3^m = 3^m(3^3 - 1)$
$3^3 - 1 = 26$
$= 26 \times 3^m$, which is a multiple of 26
Answer: 26×3^m

7 Nature from the gradient either side

$y = 2x^3 + 3x^2 - 12x - 4$
(a) Show that there is a stationary point at $x = 1$.
(b) By finding the gradient when $x = \frac{1}{2}$ and when $x = \frac{3}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 6x^2 + 6x - 12$, so when $x = 1$: $\frac{dy}{dx} = 0$
$x = \frac{1}{2}$: gradient $= -\frac{15}{2}$ (negative)
$x = \frac{3}{2}$: gradient $= \frac{21}{2}$ (positive)
The gradient goes from negative to 0 to positive, so $(1, -11)$ is a minimum
Answer: (b) minimum

8 3D trigonometry: pyramid

VABCD is a pyramid. The base ABCD is a square with sides of 14 cm.
The vertex V is 15 cm vertically above the centre O of the base.
M is the midpoint of BC.
Work out the angle between the face VBC and the base. (Calculator)

OM is perpendicular to BC, with OM = ½ × 14 = 7 cm
VM is perpendicular to BC, so the angle between the face and the base is angle VMO
$\tan VMO = \frac{VO}{OM} = \frac{15}{7}$
Angle = 65.0°
Answer: 65.0°

9 Product of brackets: tangent gradient

$y = x(x - 4)(2x + 2)$

Work out the gradient of the curve at the point where $x = 2$.

Expand: $y = 2x^3 - 6x^2 - 8x$

$\frac{dy}{dx} = 6x^2 - 12x - 8$

$x = 2$: $\frac{dy}{dx} = -8$

Answer: -8

10 Bounds of roots

The volume of a cube is 191 cm^3, correct to the nearest cm^3.

Work out the upper bound of the edge length of the cube.

Give your answer correct to 3 significant figures. (Calculator)

$190.5 \leq$ volume < 191.5

Upper bound of edge $= \sqrt[3]{191.5}$

$= 5.76399... = 5.76$ (3 s.f.)

Answer: 5.76 cm