Day 325 **IGCSE 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x+1}{2} - \frac{x-6}{8} = 2$

Multiply every term by 8: $4(x + 1) - (x - 6) = 16$
Expand: $3x + 10 = 16$
$3x = 6$
$x = 2$
Answer: $x = 2$

2 Differentiating polynomials

$y = x^3 - 3x^2 - 4x + 3$

Find $\frac{dy}{dx}$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$
$x^3 \to 3x^2$, $-3x^2 \to -6x$, $-4x \to -4$
$\frac{dy}{dx} = 3x^2 - 6x - 4$
Answer: $\frac{dy}{dx} = 3x^2 - 6x - 4$

3 Simplifying surds

Write $\sqrt{63} - \sqrt{28} + \sqrt{175}$ in the form $a\sqrt{7}$
where a is an integer.

$\sqrt{63} = 3\sqrt{7}$; $\sqrt{28} = 2\sqrt{7}$; $\sqrt{175} = 5\sqrt{7}$
$(3 - 2 + 5)\sqrt{7} = 6\sqrt{7}$
Answer: $6\sqrt{7}$

4 Point with a given gradient (quadratic)

A curve has equation $y = -2x^2 - 4x - 3$.
The gradient at the point P on the curve is -4.
Find the coordinates of P.

$\frac{dy}{dx} = -4x - 4$
$-4x - 4 = -4$, so $-4x = 0$
$x = 0$
$y = -3$
Answer: (0, −3)

5 Estimating the mean from a histogram table

The table gives information about the times of some people.
Work out an estimate for the mean time.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 4, 40, 30, 40
Mid-points: 5, 20, 45, 80
$\Sigma fm = 4\times5 + 40\times20 + 30\times45 + 40\times80 = 5370$, $\Sigma f = 114$
Mean = 5370 ÷ 114 = 47.1
Answer: 47.1

6 Inverse and repeated functions

f(x) = 3x − 6.

(a) Find $f^{-1}(3)$.

(b) Express ff(x) in the form px + q.

(a) Solve 3x − 6 = 3: 3x = 9

x = 3 so $f^{-1}(3) = 3$

(b) ff(x) = 3(3x − 6) − 6 = 9x − 24

Answer: (a) 3 (b) ff(x) = 9x − 24

7 Iteration: rearranging equations

(a) Show that the equation $x^3 + 4x - 33 = 0$ has a solution between x = 2 and x = 3.

(b) Show that the equation can be rearranged to give $x = \sqrt[3]{33 - 4x}$

(a) Let $f(x) = x^3 + 4x - 33$. $f(2) = 2^3 + 4 \times 2 - 33 = -17$ and $f(3) = 3^3 + 4 \times 3 - 33 = 6$

f(2) is negative and f(3) is positive. There is a change of sign, so there is a solution between 2 and 3

(b) Rearrange $x^3 + 4x - 33 = 0$: $x^3 = 33 - 4x$

Take the cube root of both sides: $x = \sqrt[3]{33 - 4x}$

Answer: Shown

8 Bounds: finding an unknown length

A rectangle has area 25 cm^2, correct to 2 significant figures.

Its length is 5.200000000000000177635683940025046467781066894531 25 cm, correct to 2 significant figures.

Work out the lower bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length

Area: 24.5 ≤ A < 25.5. Length: 5.150000000000000177635683940025046467781066894531 25 ≤ l < 5.250000000000000177635683940025046467781066894531 25

LB width = LB area ÷ UB length = 24.5 ÷ 5.250000000000000177635683940025046467781066894531 25 = 4.67

Answer: 4.67 cm

9 Stationary points of y = ax + b/x

$y = -4x - \frac{36}{x}$

Find the coordinates of the stationary points and determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = -4 + \frac{36}{x^2}$

$\frac{dy}{dx} = 0$: $\frac{36}{x^2} = 4$, so $x^2 = 9$ and x = 3 or x = −3

y = −24 when x = 3 and y = 24 when x = −3

$\frac{d^2y}{dx^2} = -\frac{72}{x^3}$

x = 3: $\frac{d^2y}{dx^2} = -\frac{8}{3}$ so maximum; x = −3: $\frac{d^2y}{dx^2} = \frac{8}{3}$ so minimum

Answer: (3, −24) maximum; (−3, 24) minimum

10 Exact trigonometric ratios from a given ratio

x is an acute angle and $\cos x = \frac{4}{13}$.

(a) Find the exact value of sin x.

(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\sin^2 x + \cos^2 x = 1$, so $\sin^2 x = 1 - (\frac{4}{13})^2 = \frac{153}{169}$

x is acute, so sin x is positive: $\sin x = \frac{3\sqrt{17}}{13}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{3\sqrt{17}}{13} \div \frac{4}{13} = \frac{3\sqrt{17}}{4}$

Answer: (a) $\frac{3\sqrt{17}}{13}$ (b) $\frac{3\sqrt{17}}{4}$