Day 324 **IGCSE 10-a-day: Model Solutions**

1 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 40 cm. (Calculator)
The outer radius is 6 cm and the inner radius is 5 cm.
The density of the metal is 2.7 g/cm^3.
Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal = $\pi \times 6^2 \times 40 - \pi \times 5^2 \times 40 = \pi(36 - 25) \times 40$
$= 1382.3$ cm^3
Mass = density × volume = $2.7 \times 1382.3 = 3732.2$ g
$= 3.73$ kg (3 s.f.)
Answer: 3.73 kg

2 Expand then differentiate

$y = (x + 6)(x - 5)$
Find $\frac{dy}{dx}$.

Expand: $y = x^2 + x - 30$
$x^2 \to 2x$, $x \to 1$
$\frac{dy}{dx} = 2x + 1$
Answer: $\frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{2x + 1}$

3 Tests of divisibility

The 5-digit number 737k2 is a multiple of 12.
k is a single digit.
Find all the possible values of k.

A multiple of 12 is a multiple of 3 and a multiple of 4
Digit sum = 7 + 3 + 7 + 2 + k = 19 + k
This must be a multiple of 3, so k = 2 or 5 or 8
Last two digits k2 must be a multiple of 4: k = 5
Both conditions give k = 5
Answer: k = 5

4 Trigonometric equations

Solve $2 \cos x° + 3 = 5$ for $0 \leq x \leq 360$

Rearrange: $2 \cos x° = 2$, so $\cos x° = 1$
From the graph of $y = \cos x°$, $\cos x° = 1$ when $x = 0$ or 360
Answer: x = 0, 360

5 Sine rule: finding an angle

Angle B is acute.
Work out the size of angle ABC.
Give your answer correct to 1 decimal place. (Calculator)

$\frac{\sin B}{AC} = \frac{\sin A}{BC}$
$\sin B = \frac{12.4 \times \sin 98°}{16}$
$B = \sin^{-1}(0.7675) = 50.1°$
Answer: 50.1°

6 Inverse proportion in context

24 painters can paint a hall in 24 days.
After 3 days, 3 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = $24 \times 24 = 576$ painter-days
Work done = $24 \times 3 = 72$, so work left = 504
Workers left = $24 - 3 = 21$
Time = $504 \div 21 = 24$ days
Answer: 24 days

7 Show that the gradient is always positive

A curve has equation $y = x^3 + 3x^2 + 4x + 6$.
Show that y is an increasing function for all values of x.

$\frac{dy}{dx} = 3x^2 + 6x + 4$

Discriminant = $6^2 - 4 \times 3 \times 4 = -12$

The discriminant is negative, so $\frac{dy}{dx} = 0$ has no real solutions

The coefficient of x^2 is positive, so $\frac{dy}{dx} > 0$ for all x

So y is increasing for all values of x
Answer: Discriminant = $-12 < 0$, so $dy/dx > 0$ for all x

8 Ratio and mixtures

Tin A holds red paint and white paint in the ratio 4 : 2.
Tin B holds red paint and white paint in the ratio 5 : 2.
36 litres from tin A and 14 litres from tin B are mixed.
Work out the ratio of red paint to white paint in the mixture.

From A: red paint = $36 \times \frac{2}{3} = 24$ litres, white paint = 12 litres

From B: red paint = $14 \times \frac{5}{7} = 10$ litres, white paint = 4 litres

red paint: $24 + 10 = 34$ and white paint: $12 + 4 = 16$
34 : 16 = 17 : 8
Answer: 17 : 8

9 Number of solutions of f(x) = k

The diagram shows a sketch of the curve $y = x^3 - 9x^2 + 24x + 8$ and the line $y = k$.
The line meets the curve at three different points.
Find the range of possible values of k.

$\frac{dy}{dx} = 3x^2 - 18x + 24 = 0$ gives $x = 2$ and $x = 4$

$y = 28$ at $x = 2$ and $y = 24$ at $x = 4$
The line crosses three times when it lies between the two turning-point values
$24 < k < 28$
Answer: $24 < k < 28$

10 Show that lines are parallel

In triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$.
M is on OA and N is on OB, with OM : MA = ON : NB = 2 : 1.
(a) Show that MN is parallel to AB.
(b) Write down the ratio MN : AB.

(a) $\mathbf{OM} = \frac{2}{3}\mathbf{a}$ and $\mathbf{ON} = \frac{2}{3}\mathbf{b}$

$\mathbf{MN} = \mathbf{MO} + \mathbf{ON} = -\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$

$\mathbf{b} - \mathbf{a} = \mathbf{AB}$, so $\mathbf{MN} = \frac{2}{3}\mathbf{AB}$: a multiple of $\mathbf{AB}$, so MN is parallel to AB

(b) MN : AB = 2 : 3

Answer: (b) 2 : 3