Day 323 **IGCSE 10-a-day: Model Solutions**

1 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where

$s = 3t^2 + t + 7 + \frac{16}{t}$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when t = 2.

$v = \frac{ds}{dt} = 6t + 1 - 16t^{-2}$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6 + 32t^{-3}$

When t = 2: v = 9 and a = 10

Answer: (a) 9 m/s (b) 10 m/s^2

2 Direct proportion formulae

E is directly proportional to the square root of v.
When v = 4, E = 6.
(a) Find a formula for E in terms of v.
(b) Work out the positive value of v when E = 12.

(a) $E = k\sqrt{v}$
$6 = k \times 2$, so $k = 6 \div 2 = 3$
$E = 3\sqrt{v}$
(b) $12 = 3\sqrt{v}$, so $\sqrt{v} = 12 \div 3 = 4$, giving v = 16

Answer: (a) $E = 3\sqrt{v}$ (b) v = 16

3 Solve f(x) = g(x)

$f(x) = 3x^2 - 1$ and $g(x) = -6x + 8$.
Solve f(x) = g(x).

$3x^2 - 1 = -6x + 8$
$3x^2 + 6x - 9 = 0$
$3(x + 3)(x - 1) = 0$
$x = -3$ or $x = 1$

Answer: x = −3 or x = 1

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 80°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: 360 − 90 − 90 − 80 = 100°
(b) The angle at the circumference is half the angle at the centre: 100 ÷ 2 = 50°

Answer: (a) 100° (b) 50°

5 Laws of indices with algebra

Simplify fully $\frac{7x^2y^6 \times 6x^2y^2}{2xy^4}$

Give your answer in the form kx^ay^b

Numbers: 7 × 6 ÷ 2 = 21
Powers of x: 2 + 2 − 1 = 3
Powers of y: 6 + 2 − 4 = 4
$= 21x^3y^4$

Answer: $21x^3y^4$

6 Gradient function from a given point

A curve has equation $y = -2x^3 + 3x^2 + 36x + 4$.
The point A $(-2, k)$ lies on the curve. Work out the value of k and the gradient of the curve at A.

Substitute $x = -2$ into the equation of the curve
$y = -40$ when $x = -2$, so $k = -40$
$\frac{dy}{dx} = -6x^2 + 6x + 36$
At A: gradient = 0
Answer: k = −40, gradient = 0

7 Error intervals

The distance to a village is 0.55 km, correct to 2 decimal places.
Write down the error interval for the distance to a village, using d.

The value has been rounded to the nearest 0.01
Half of this is 0.005, so go 0.005 below and 0.005 above
$0.55 - 0.005 = 0.545$ and $0.55 + 0.005 = 0.555$
$0.545 \leq d < 0.555$
Answer: $0.545 \leq d < 0.555$

8 Winning a match

Liam and Leila play a match. The first to win 2 chess games wins the match.

The probability that Liam wins any one of the chess games is $\frac{2}{3}$.

Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two

P(Liam wins then loses) = $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

P(Liam loses then wins) = $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

Total = $2 \times \frac{2}{9} = \frac{4}{9}$

Answer: $\frac{4}{9}$

9 Area of triangle from a tangent

The tangent to the curve $y = x^2 + 4x - 4$ at the point where $x = 2$ meets the axes at A and B.
Work out the area of triangle OAB, where O is the origin.

$y = 8$ and $\frac{dy}{dx} = 2x + 4$, so the gradient is 8

Tangent: $y = 8x - 8$
y-axis: $(0, -8)$. x-axis: $0 = 8x - 8$ so $x = 1$

Area = $\frac{1}{2} \times 8 \times 1$

= 4
Answer: 4 square units

10 Linear programming: vertices

x and y satisfy $x \geq 0$, $y \geq 0$,
$4x + 2y \leq 40$ and $5x + 5y \leq 75$.
Find the greatest value of $P = 5x + 3y$ and the values of x and y that give it.

Draw the lines. The feasible region has vertices (0, 0), (0, 15), (5, 10), (10, 0)
Values of P: (0, 0): 0, (0, 15): 45, (5, 10): 55, (10, 0): 50
Greatest value of P is 55 at (5, 10)
Answer: P = 55 when x = 5 and y = 10