Day 322 **IGCSE 10-a-day: Model Solutions**

1 Second derivative

$y = 2x^5 - 8x^4 - 4x^2$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Work out the value of $\frac{d^2y}{dx^2}$ when $x = -2$.

$\frac{dy}{dx} = 10x^4 - 32x^3 - 8x$

$\frac{d^2y}{dx^2} = 40x^3 - 96x^2 - 8$

$x = -2$: $\frac{d^2y}{dx^2} = -712$

Answer: (a) $\frac{dy}{dx} = 10x^4 - 32x^3 - 8x$, $\frac{d^2y}{dx^2} = 40x^3 - 96x^2 - 8$ (b) -712

2 Weighted percentages

The ratio of boys to girls in a school is 4 : 1.
80% of the boys and 50% of the girls walk to school.
What percentage of all of these students walk to school?

Imagine 100 students in total: 80 boys and 20 girls
80% of 80 = 64 and 50% of 20 = 10
Together: 74 out of 100
74 ÷ 100 × 100 = 74%
Answer: 74%

3 Maximum revenue

A company sells x thousand items each month. The price of each item is £40 − 4x.
The monthly revenue is £R thousand.
(a) Show that $R = 40x - 4x^2$.
(b) Find the maximum monthly revenue, showing that it is a maximum.

$R = x \times (40 - 4x) = 40x - 4x^2$

$\frac{dR}{dx} = -8x + 40 = 0$ gives $x = 5$

$\frac{d^2R}{dx^2} = -8 < 0$, so a maximum

$R = 40 \times 5 - 4 \times 5^2 = 100$
Answer: £100 thousand

4 Similar shapes

Two cones, P and Q, are mathematically similar.
The volume of P is 16 cm^3 and the volume of Q is 250 cm^3.
The curved surface area of P is 24 cm^2.
Work out the curved surface area of Q.

Ratio of volumes = 16 : 250 = $2^3 : 5^3$
So the ratio of lengths is 2 : 5

Ratio of areas = $2^2 : 5^2$, so scale factor = $\frac{25}{4}$

$24 \times \frac{25}{4} = 150$ cm^2

Answer: 150 cm^2

5 Area of a triangle: ½ab sin C

The area of triangle ABC is 12.1 cm^2.
Angle C is acute. Work out the size of angle C.
Give your answer correct to 1 decimal place. (Calculator)

Area = ½ × a × b × sin C
12.1 = ½ × 14.5 × 10.1 × sin C

$\sin C = \frac{24.2}{146.45} = 0.1652$

C = 9.5°
Answer: 9.5°

6 Compound interest

Fatima puts £1,500 in a savings account.
Compound interest is paid at 4% in year 1, 6% in year 2 and 5% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.04, 1.06, 1.05
£1,500 × 1.04 × 1.06 × 1.05
= £1,736.28
Answer: £1,736.28

7 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 3**a** + 4**b**, OQ = 6**a** − 8**b** and OR = 10**a** − 24**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 3**a** − 12**b**
QR = OR − OQ = 4**a** − 16**b**

$QR = \frac{4}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 4
Answer: (b) 3 : 4

8 Solve f′(x) = 0 with a reciprocal term

$f(x) = 3x^3 + \frac{144}{x}$

Find the values of x for which f′(x) = 0.

$f'(x) = 9x^2 - \frac{144}{x^2}$

$f'(x) = 0$: $9x^2 = \frac{144}{x^2}$

$x^4 = 16$
x = 2 or x = −2
Answer: x = 2 or x = −2

9 Solving AX = B

$A = \begin{pmatrix} 4 & 1 \\ 3 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 19 \\ 21 \end{pmatrix}$.

AX = B, where X is a 2 × 1 matrix.

Find X.

$X = A^{-1}B$ (multiply both sides on the left by A^{-1})

$A^{-1} = \frac{1}{9}\begin{pmatrix} 3 & -1 \\ -3 & 4 \end{pmatrix}$

$X = \frac{1}{9}\begin{pmatrix} 3 & -1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 19 \\ 21 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Answer: $\mathbf{X = \begin{pmatrix} 4 \\ 3 \end{pmatrix}}$

10 Equations with algebraic fractions

Solve $\frac{3}{x+1} + \frac{2}{x-2} = 2$

Multiply every term by $(x + 1)(x - 2)$: $3(x - 2) + 2(x + 1) = 2 \times (x + 1)(x - 2)$

Expand and rearrange so that one side is 0: $2x^2 - 7x = 0$

Factorise: $x(2x - 7) = 0$

$x = 0$ or $x = \frac{7}{2}$

Answer: $\mathbf{x = 0}$ **or** $\mathbf{x = \frac{7}{2}}$