Day 321 **IGCSE 10-a-day: Model Solutions**

1 Arithmetic sequences

The 3rd term of an arithmetic sequence is 23.
The 6th term of the sequence is 50.
(a) Find an expression for the nth term of the sequence.
(b) Is 301 a term of the sequence? You must show your working.

(a) Common difference = $(50 - 23) \div (6 - 3) = 9$
nth term = $9n + c$. For n = 3: $9 \times 3 + c = 23$, so $c = -4$
nth term = $9n - 4$

(b) Solve $9n - 4 = 301$: $9n = 305$, so $n = \frac{305}{9}$

n is not a whole number, so 301 is not a term
Answer: (a) $9n - 4$ (b) No

2 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by
$h = -4t^2 + 40t + 15$
(a) Work out the greatest height reached by the ball.
(b) Work out the velocity of the ball when t = 12. State whether it is moving up or down.

$\frac{dh}{dt} = -8t + 40 = 0$ gives $t = 5$

$\frac{d^2h}{dt^2} = -8 < 0$ so maximum. h = 115

When t = 12: $\frac{dh}{dt} = 40 - 8 \times 12 = -56$

Answer: (a) 115 m (b) −56 m/s, moving down

3 Matrix of a transformation

A transformation T maps (1, 0) to (4, 2) and (0, 1) to (−1, 0).
(a) Write down the matrix that represents T.
(b) Find the image of the point (−3, 3) under T.

(a) The images of (1, 0) and (0, 1) form the columns of the matrix

Matrix = $\begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} -3 \\ 3 \end{pmatrix} = \begin{pmatrix} -15 \\ -6 \end{pmatrix}$

Image is (−15, −6)

Answer: (a) $\begin{pmatrix} \mathbf{4} & \mathbf{-1} \\ \mathbf{2} & \mathbf{0} \end{pmatrix}$ (b) (−15, −6)

4 At least one

The probability that a light bulb in a hallway fails in any month is $\frac{1}{4}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 5 months.
Give your answer correct to 3 significant figures. (Calculator)

P(never) = $(1 - \frac{1}{4})^5 = \frac{3}{4}^5 = 0.2373046875$

P(at least once) = 1 − P(never) = 1 − 0.2373046875
= 0.763

Answer: 0.763

5 Differentiating brackets with square roots

$y = (\sqrt{x} + 5)(\sqrt{x} + 4)$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Expand: $y = x + 9x^{1/2} + 20$

$x \to 1,\ 9x^{1/2} \to \frac{9}{2}x^{-1/2}$

$\frac{dy}{dx} = 1 + \frac{9}{2}x^{-1/2}$

Answer: $\mathbf{\frac{dy}{dx} = 1 + \frac{9}{2}x^{-1/2}}$

6 Recasting solids

A solid cylinder of radius 4 cm and height 12 cm is melted down.
The metal is used to make identical solid spheres of radius 2 cm.
Work out the number of spheres that can be made.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of cylinder $= \pi \times 4^2 \times 12 = 192\pi$

Volume of one sphere $= \frac{4}{3} \times \pi \times 2^3 = \frac{32}{3}\pi$

Number of spheres $= 192 \div \frac{32}{3} = 18$

Answer: 18

7 Show that a line is a tangent

The line $y = -6x + k$ is a tangent to the curve $y = 3x^2 - 6x + 3$.
Find the value of the constant k.

$\frac{dy}{dx} = 6x - 6 = -6$, so $x = 0$

$y = 3$ when $x = 0$

The line passes through (0, 3): $3 = -6 \times 0 + k$, so $k = 3$

Answer: k = 3

8 Radius from chord and height of arc

A circular arc has a chord AB of length 20 cm.
The greatest distance between the chord and the arc is 2 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so $OM = r - 2$

$AM = 20 \div 2 = 10$. Pythagoras in triangle OMA: $r^2 = (r - 2)^2 + 10^2$

$r^2 = r^2 - 4r + 4 + 100$

$4r = 104$, so $r = 26$ cm

Answer: 26 cm

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9 Simultaneous equations with an inverse matrix

$M = \begin{pmatrix} -1 & 5 \\ 2 & -1 \end{pmatrix}$.

(a) Find M^{-1}.

(b) Use your answer to (a) to solve

$-x + 5y = 11$

$2x - y = 5$

(a) det M = -9 so $M^{-1} = -\frac{1}{9}\begin{pmatrix} -1 & -5 \\ -2 & -1 \end{pmatrix}$

(b) The equations are $\begin{pmatrix} -1 & 5 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{9}\begin{pmatrix} -1 & -5 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 11 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Answer: (a) $\mathbf{-\frac{1}{9}\begin{pmatrix} -1 & -5 \\ -2 & -1 \end{pmatrix}}$ **(b) x = 4, y = 3**

10 Expanding brackets with surds

Show that $(5 - 2\sqrt{7})(5 + 2\sqrt{7})$ is an integer.

Use the difference of two squares: $(5 - 2\sqrt{7})(5 + 2\sqrt{7}) = 25 - (2\sqrt{7})^2$

$= 25 - 28 = -3$

Answer: -3