Day 320 **IGCSE 10-a-day: Model Solutions**

1 Line and curve intersections

Solve the simultaneous equations

$y = x^2 - 4x - 1$

$y = -8x - 1$

Substitute: $x^2 - 4x - 1 = -8x - 1$

Rearrange: $x^2 + 4x = 0$

Factorise: $(x + 4)x = 0$, so $x = -4$ or $x = 0$

When $x = -4$: $y = -8 \times (-4) - 1 = 31$

When $x = 0$: $y = -8 \times 0 - 1 = -1$

Answer: $x = -4$, $y = 31$ and $x = 0$, $y = -1$

2 Angles with algebra

ABC is an isosceles triangle with AB = AC.

Angle BAC = $(5x - 5)°$

Angle ABC = $(3x - 23)°$

(a) Work out the value of x.

(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°

$(5x - 5) + 2(3x - 23) = 180$

$11x - 51 = 180$, so $11x = 231$

$x = 21$

(b) Angle BAC = $5 \times 21 - 5 = 100°$

Answer: (a) $x = 21$ (b) 100°

3 Inclusion and exclusion with two sets

There are 39 students.

13 of them study French and 15 of them study Spanish.

20 of them do neither.

(a) Work out how many do both.

(b) One of the students is chosen at random. Find the probability that they only study French.

Number in A or B = $39 - 20 = 19$

(a) $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 13 + 15 - 19 = 9$

(b) Number who only study French = $13 - 9 = 4$

Probability = $\frac{4}{39}$

Answer: (a) 9 (b) $\frac{4}{39}$

4 Gradient from a tangent

The diagram shows part of the curve $y = -2x^2 + 3x + 1$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)

(a) Use the two marked points on the tangent to work out its gradient.

(b) Use differentiation to find the x coordinate of P.

Gradient = $\frac{-6 - 9}{3 - 0} = -5$

$\frac{dy}{dx} = -4x + 3$

$-4x + 3 = -5$, so $x = 2$

Answer: (a) -5 (b) $x = 2$

5 Three-part ratios

Zane, Beth and Uma share some money.
Zane : Beth = 4 : 2
Beth : Uma = 6 : 3
(a) Work out the ratio Zane : Beth : Uma.
(b) They share £63 altogether. How much money does Uma get?

(a) Make the Beth part the same: 4 : 2 = 12 : 6 and 6 : 3 = 6 : 3
Zane : Beth : Uma = 12 : 6 : 3 = 4 : 2 : 1
(b) Total parts = 7, so 1 part = 63 ÷ 7 = 9
Uma gets 1 × 9 = £9
Answer: (a) 4 : 2 : 1 (b) £9

6 Powers of standard form

Work out $(3.3 \times 10^{-2})^3$
Give your answer in standard form.

$(3.3 \times 10^{-2})^3 = 3.3^3 \times (10^{-2})^3$
$3.3^3 = 35.937$ and $(10^{-2})^3 = 10^{-6}$
$35.937 \times 10^{-6} = 3.5937 \times 10^{-5}$
Answer: 3.5937×10^{-5}

7 Turning point after a transformation

The graph of y = f(x) has a maximum point at (−5, −9).
Write down the coordinates and the type of turning point on each graph.
(a) y = −f(x)
(b) y = f(x − 1)

(a) y = −f(x): reflection in the x-axis turns a maximum into a minimum
(−5, 9), a minimum
(b) y = f(x − 1): x-coordinate increases by 1
(−4, −9), a maximum
Answer: (a) (−5, 9) minimum (b) (−4, −9) maximum

8 Find constants from a gradient

A curve has equation $y = ax^2 + \frac{b}{x}$, where a and b are constants.

The point (1, −7) lies on the curve and the gradient of the curve at this point is −2.
Find the values of a and b.

$\frac{dy}{dx} = 2ax - bx^{-2} = 2ax - \frac{b}{x^2}$
Point on curve: a + b = −7. Gradient: 2a − b = −2
Solving simultaneously gives a = −3 and b = −4
Answer: a = −3, b = −4

9 Probability and quadratic equations

A bag contains some counters. 4 of them are green and the rest are white.
Two counters are taken at random without replacement.

The probability that both are green is $\frac{1}{11}$.

Work out how many counters are white.

Let there be n counters altogether
P(both green) $= \frac{4}{n} \times \frac{3}{n-1} = \frac{12}{n(n-1)}$
$\frac{12}{n(n-1)} = \frac{1}{11}$, so $n(n-1) = 132$, giving $n^2 - n - 132 = 0$
$(n - 12)(n + 11) = 0$, so n = 12 (n cannot be negative)
Number of counters that are white = 12 − 4 = 8
Answer: 8

10 Normal meets the x-axis

The normal to the curve $y = x^2 + 3x + 4$ at the point where $x = -2$ meets the x-axis at N.
Find the coordinates of N.

$y = 2$ and gradient of curve $= -1$
Gradient of normal $= 1$
$y - 2 = x + 2$
Put $y = 0$: $-2 = x + 2$
$x = -4$
Answer: $(-4, 0)$