Day 32 **IGCSE 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 14 times the size of each exterior angle.
Work out the number of sides of the polygon.

Let each exterior angle be e. Then each interior angle is 14e.
Interior angle + exterior angle = 180°, so 14e + e = 180°
e = 180° ÷ 15 = 12°
Number of sides = 360° ÷ 12° = 30
Answer: 30

2 Exact trigonometric values

Work out the exact value of 6 sin 45° ÷ cos 30°.
Give your answer in its simplest form, with no decimals.

$\sin 45° = \frac{\sqrt{2}}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$

Substitute and simplify (rationalise any denominator)
$= 2\sqrt{6}$
Answer: $2\sqrt{6}$

3 Second derivative

$y = 2x^2 + \frac{8}{x^2}$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Work out the value of $\frac{d^2y}{dx^2}$ when $x = -2$.

$\frac{dy}{dx} = 4x - 16x^{-3}$

$\frac{d^2y}{dx^2} = 4 + 48x^{-4}$

$x = -2$: $\frac{d^2y}{dx^2} = 7$

Answer: (a) $\frac{dy}{dx} = 4x - \frac{16}{x^3}$, $\frac{d^2y}{dx^2} = 4 + \frac{48}{x^4}$ (b) 7

4 Pressure

A concrete slab in the shape of a cuboid weighs 120 N.
Its smallest face has area 100 cm^2. Its largest face has area 2,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 100 cm^2 = 0.01 m^2 and 2,000 cm^2 = 0.2 m^2
(a) Greatest pressure is on the smallest face: 120 ÷ 0.01 = 12,000 N/m^2
(b) Least pressure is on the largest face: 120 ÷ 0.2 = 600 N/m^2
Answer: (a) 12,000 N/m^2 (b) 600 N/m^2

5 HCF and LCM

A = $2^2 \times 3^3 \times 7$

B = $2^3 \times 7^2$

Write down, as a product of prime factors,

(a) the HCF of A and B

(b) the LCM of A and B

HCF: take the lowest power of each prime that appears in both numbers

= $2^2 \times 7$

LCM: take the highest power of every prime that appears

= $2^3 \times 3^3 \times 7^2$

Answer: (a) $2^2 \times 7$ (b) $2^3 \times 3^3 \times 7^2$

6 Tangent perpendicular to a line

The tangent at the point P on the curve $y = 3x^2 - x$ is perpendicular to the line $x - y = 9$.

Find the coordinates of P.

The line is $y = x - 9$, so its gradient is 1

The tangent has gradient -1

$\frac{dy}{dx} = 6x - 1 = -1$ so $x = 0$

$y = 0$

Answer: (0, 0)

7 Composite functions: fg and gf

$f(x) = 4x + 2$ and $g(x) = x^2 - 1$

Solve $fg(x) = gf(x)$

$fg(x) = f(x^2 - 1) = 4x^2 - 2$

$gf(x) = g(4x + 2) = (4x + 2)^2 - 1 = 16x^2 + 16x + 3$

Set them equal and rearrange: $12x^2 + 16x + 5 = 0$

Factorise: $(2x + 1)(6x + 5) = 0$

$x = -\frac{5}{6}$ or $x = -\frac{1}{2}$

Answer: $x = -\frac{5}{6}$ or $x = -\frac{1}{2}$

8 A squared in terms of A and I

$A = \begin{pmatrix} 5 & 6 \\ -4 & 3 \end{pmatrix}$.

$A^2 = pA + qI$, where I is the 2×2 identity matrix.

Find the value of p and the value of q.

$A^2 = \begin{pmatrix} 1 & 48 \\ -32 & -15 \end{pmatrix}$

Top right entries: $48 = p \times 6$, so $p = 8$

Top left entries: $1 = 8 \times 5 + q$, so $q = -39$

Check bottom right: $8 \times 3 - 39 = -15$

Answer: $p = 8$, $q = -39$

9 Rectangle under a parabola

The diagram shows the curve $y = 3 - x^2$. Rectangle ABCD has A, B on the x-axis and C, D on the curve. B is (x, 0).
(a) Show that the area of ABCD is $-2x^3 + 6x$.
(b) Find the maximum area of ABCD, showing that it is a maximum.

AB = 2x and BC = $3 - x^2$, so area = $2x(3 - x^2) = -2x^3 + 6x$

$\frac{dA}{dx} = -6x^2 + 6 = 0$, so $x^2 = 1$ and $x = 1$ ($x > 0$)

$\frac{d^2A}{dx^2} = -12x < 0$ when $x = 1$, so a maximum

Area = $6 \times 1 - 2 \times 1^3 = 4$

Answer: 4

10 Equations with indices

Solve $5^{2x+1} = \frac{1}{25}$

Write the right-hand side as a power of 5: $\frac{1}{25} = 5^{-2}$

Equate the powers: $2x + 1 = -2$

$2x = -3$

$x = -\frac{3}{2}$

Answer: $x = -\frac{3}{2}$