Day 319 **IGCSE 10-a-day: Model Solutions**

1 Factorising fully

Factorise fully $4x^2 + 28x + 24$

Take out the common factor 4: $4(x^2 + 7x + 6)$
Factorise the quadratic: $4(x + 6)(x + 1)$
Answer: $4(x + 6)(x + 1)$

2 Tests of divisibility

The 4-digit number 685k is a multiple of 12.
k is a single digit.
Find all the possible values of k.

A multiple of 12 is a multiple of 3 and a multiple of 4
Digit sum = 6 + 8 + 5 + k = 19 + k
This must be a multiple of 3, so k = 2 or 5 or 8
Last two digits 5k must be a multiple of 4: k = 2
Both conditions give k = 2
Answer: k = 2

3 Bearings with the sine and cosine rules

A plane leaves an airport P and travels 49 km on a bearing of 045° to Q. (Calculator)
It then travels 113 km on a bearing of 125° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 045° + 180° = 225°
Angle PQR = 100° (the difference between the bearings 225° and 125°)
$PR^2 = 49^2 + 113^2 - 2 \times 49 \times 113 \times \cos 100° = 17093$, so PR = 130.7 km

(b) $\cos(QPR) = \frac{49^2 + PR^2 - 113^2}{2 \times 49 \times PR}$, so angle QPR = 58.3°

R is clockwise of the direction PQ, so the bearing is 045° + 58° = 103°
Answer: (a) 130.7 km (b) 103°

4 Gradient from a tangent

The diagram shows part of the curve $y = -2x^2 - 2x - 2$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)
(a) Use the two marked points on the tangent to work out its gradient.
(b) Use differentiation to find the x coordinate of P.

Gradient = $\frac{-4 - 2}{1 - (-2)} = -2$

$\frac{dy}{dx} = -4x - 2$
$-4x - 2 = -2$, so x = 0
Answer: (a) -2 (b) x = 0

5 Percentages of percentages

In a group of visitors, 65% are adults and the rest are children.
45% of the adults and 30% of the children buy a souvenir.
What percentage of all the visitors buy a souvenir?

Take 100 visitors: 65 are adults and 35 are children
45% of 65 = 29.25 and 30% of 35 = 10.5
29.25 + 10.5 = 39.75 out of 100
Answer: 39.75%

6 Average and instantaneous velocity

A particle moves in a straight line. Its displacement from O after t seconds is s metres, where $s = 3t^2 - 8t + 1$

(a) Work out the average velocity of the particle between t = 0 and t = 4.

(b) Find the time at which the velocity equals this average velocity.

s(0) = 1 and s(4) = 17

Average velocity = $\frac{17 - 1}{4 - 0} = 4$

$v = \frac{ds}{dt} = 6t - 8$

$6t - 8 = 4$ gives $t = 2$

Answer: (a) 4 m/s (b) t = 2 s

7 Average speed

A bus travels the first half of a journey at 30 km/h.

It travels the second half of the same journey at 45 km/h.

Work out the average speed for the whole journey.

Let each half of the journey be 90 km (a common multiple of 30 and 45)

Time for first half = 90 ÷ 30 = 3 hours

Time for second half = 90 ÷ 45 = 2 hours

Average speed = total distance ÷ total time = 180 ÷ 5 = 36 km/h

Answer: 36 km/h

8 Linear programming: vertices

x and y satisfy $x \geq 0$, $y \geq 0$,

$6x + 6y \leq 78$ and $2x + 6y \leq 66$.

Find the greatest value of $P = 4x + 9y$ and the values of x and y that give it.

Draw the lines. The feasible region has vertices (0, 0), (0, 11), (3, 10), (13, 0)

Values of P: (0, 0): 0, (0, 11): 99, (3, 10): 102, (13, 0): 52

Greatest value of P is 102 at (3, 10)

Answer: P = 102 when x = 3 and y = 10

9 Cylinder of fixed surface area

A closed cylinder (a container) has radius r cm, height h cm and total surface area 486π cm^2.

(a) Show that its volume, V cm^3, is $V = 243\pi r - \pi r^3$.

(b) Find the maximum value of V in terms of π, and show that h = 2r when V is a maximum.

$2\pi r^2 + 2\pi rh = 486\pi$, so $h = \frac{243 - r^2}{r}$

$V = \pi r^2 h = \pi r(243 - r^2) = 243\pi r - \pi r^3$

$\frac{dV}{dr} = 243\pi - 3\pi r^2 = 0$, so $r^2 = 81$ and $r = 9$

$\frac{d^2V}{dr^2} = -6\pi r < 0$, so a maximum. $h = \frac{243 - 81}{9} = 18 = 2r$

$V = \pi \times 9^2 \times 18 = 1458\pi$

Answer: 1458π cm^3

10 Show that: trigonometric identities

Show that $6 \sin^2 x + 6 \cos^2 x + 1 \equiv 7$.

$6 \sin^2 x + 6 \cos^2 x = 6(\sin^2 x + \cos^2 x)$

$= 6 \times 1 = 6$

Add 1: 7

Answer: Shown