Day 318 **IGCSE 10-a-day: Model Solutions**

1 Histograms with an unmarked scale

The histogram shows the lengths of leaves collected from a tree.
200 of them had a length between 20 and 40 cm.
(a) How many had a length between 40 and 50 cm?
(b) How many leaves are there in total?

Frequency is proportional to the area of a bar
20 to 40: bar is 5 squares high and 20 wide, area 100, which represents 200
So 1 unit of area represents 200 ÷ 100 = 2
(a) 40 to 50: 1 × 10 = 10 units of area, so 10 × 2 = 20
(b) Total area = 10 + 100 + 10 + 160 + 80 = 360, so total = 360 × 2 = 720
Answer: (a) 20 (b) 720

2 Trigonometric equations

Solve $5 \sin x° + 3 = 3$ for $0 \leq x \leq 360$

Rearrange: $5 \sin x° = 0$, so $\sin x° = 0$
From the graph of $y = \sin x°$, $\sin x° = 0$ when $x = 0$ or 180 or 360
Answer: $x = 0, 180, 360$

3 Exponential growth and decay

A radioactive sample has a mass of 480 g. Its mass halves every 5 hours.
(a) Work out the mass after 15 hours.
(b) After how many hours will the mass first be less than 5 g?

(a) 15 hours is 3 half-lives: $480 \div 2^3 = 60$ g
(b) Halve repeatedly: 480, 240, 120, 60, 30, 15, 7.5, 3.75
3.75 is the first value below 5, after 7 half-lives, so 7 × 5 = 35 hours
Answer: (a) 60 g (b) 35 hours

4 Tangent parallel to a given line

The line $y = -4x - 1$ is parallel to the tangent at the point P on the curve $y = -2x^2 - 4x$.
Find the coordinates of P.

The line has gradient −4, so the tangent at P has gradient −4
$\frac{dy}{dx} = -4x - 4$
$-4x - 4 = -4$ gives $x = 0$
$y = 0$
Answer: (0, 0)

5 Venn diagram: shaded region notation

Write down the set notation for the region shaded in the Venn diagram.

Identify which regions are shaded, using the circles A and B
The shaded region is $A' \cap B$
Answer: $A' \cap B$

6 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the adjacent side AB and the known side is the opposite side BC, so use tan 45°
tan 45° = 1
$x = 3 \div \tan 45° = 3 \div 1$
$x = 3$
Answer: $x = 3$

7 Tangent meets the axes

The diagram shows the curve $y = x^2 + x + 7$ and the tangent at P(4, 27), meeting the axes at A and B.
(a) Find an equation of the tangent at P.
(b) Work out the area of triangle OAB.

$\frac{dy}{dx} = 2x + 1$, so at x = 4 the gradient is 9
$y - 27 = 9(x - 4)$, so $y = 9x - 9$
B: x = 0 gives y = −9. A: y = 0 gives x = 1
Area = ½ × 1 × 9 = $\frac{9}{2}$

Answer: (a) $y = 9x - 9$ (b) $\frac{9}{2}$

8 Upper and lower bounds

a = 7.7 and b = 4.6,
both correct to 1 decimal place.
Work out the upper bound of a ÷ b.
Give your answer correct to 3 significant figures.

Each value is rounded, so it can be up to 0.05 above or below
$7.65 \leq a < 7.75$; $4.55 \leq b < 4.65$
Biggest answer: biggest a ÷ smallest b = 7.75 ÷ 4.55
= 1.70330... = 1.70 to 3 significant figures

Answer: 1.70

9 Where two tangents meet

The curve C has equation $y = 2x^2 - 3x + 1$.
The points A and B on C have x coordinates −3 and 0.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = 4x - 3$. At A: gradient −15, A = (−3, 28)
Tangent at A: $y = -15x - 17$. At B: gradient −3, B = (0, 1)
Tangent at B: $y = -3x + 1$
Equate: $-15x - 17 = -3x + 1$, so $x = -\frac{3}{2}$ and $y = \frac{11}{2}$

Answer: T $\left(-\frac{3}{2}, \frac{11}{2}\right)$

10 Linear programming in context

A workshop makes x tables and y chairs each week.
Each table needs 1 hour of work and 1 metre of wood. Each chair needs 2 hours of work and 1 metre of wood.
There are 14 hours of work and 9 metres of wood available.
The income is £8 for each table and £9 for each chair.
(a) Write down two inequalities, other than $x \geq 0$, $y \geq 0$. (b) Find the greatest income.

(a) $x + 2y \leq 14$ and $x + y \leq 9$
(b) Vertices of the feasible region: (0, 0), (0, 7), (4, 5), (9, 0)
Income = 8x + 9y: 0, 63, 77, 72
Greatest income = £77 with x = 4, y = 5

Answer: (a) $x + 2y \leq 14$, $x + y \leq 9$ (b) £77