Day 317 **IGCSE 10-a-day: Model Solutions**

1 Gradient with fractional or negative powers

The equation of a curve is $y = -4x^2 - 5x - \frac{12}{x}$.

Work out the gradient of the curve at the point where $x = -2$.

$\frac{dy}{dx} = -8x - 5 + 12x^{-2}$

When $x = -2$: gradient = 14

Answer: 14

2 Surds and fractions

Simplify $\frac{1}{2+\sqrt{2}} - \frac{1}{2-\sqrt{2}}$

Give your answer with a rational denominator.

Common denominator: $(2+\sqrt{2})(2-\sqrt{2}) = 4 - 2 = 2$

Numerator: $(2-\sqrt{2}) - (2+\sqrt{2})$

$= -2\sqrt{2}$

$\frac{-2\sqrt{2}}{2} = -\sqrt{2}$

Answer: $-\sqrt{2}$

3 Speed-time graphs

The graph shows the speed of a car during 12 seconds.

(a) Work out the acceleration during the first 4 seconds.

(b) Work out the average speed over the whole 12 seconds.

(a) Acceleration = change in speed ÷ time = $(24 - 0) \div 4 = 6$ m/s^2

(b) Distance = area under the graph

Trapezium: $\frac{0+24}{2} \times 4 = 48$; rectangle: $24 \times 6 = 144$; triangle: $\frac{1}{2} \times 2 \times 24 = 24$

Total = 216 m, so average speed = $216 \div 12 = 18$ m/s

Answer: (a) 6 m/s^2 (b) 18 m/s

4 Error intervals

The number m is 38,000 correct to the nearest 1000.

Write down the error interval for m.

The rounding unit is 1000, so the half-unit is 500

Lower bound = $38000 - 500 = 37500$; upper bound = $38000 + 500 = 38500$

Answer: $37500 \leq m < 38500$

5 Fractional power functions

$f(x) = 6x^{1/2} - 4x^{-1/2}$

Find $f'(x)$ and work out the value of $f'(4)$.

$6x^{1/2} \to 3x^{-1/2}$, $-4x^{-1/2} \to 2x^{-3/2}$

$f'(x) = 3x^{-1/2} + 2x^{-3/2}$

$f'(4)$: $\sqrt{4} = 2$, so $f'(4) = \frac{7}{4}$

Answer: $f'(x) = 3x^{-1/2} + 2x^{-3/2}$, $f'(4) = \frac{7}{4}$

6 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the hypotenuse AC and the known side is the adjacent side AB, so use cos 60°

$\cos 60° = \frac{1}{2}$

$x = 6\sqrt{3} \div \cos 60° = 6\sqrt{3} \div \frac{1}{2}$

$x = 12\sqrt{3}$

Answer: $x = 12\sqrt{3}$

7 Tangent meets the axes

The diagram shows the curve $y = -x^2 + x + 4$ and the tangent at P(1, 4), meeting the axes at A and B.
(a) Find an equation of the tangent at P.
(b) Work out the area of triangle OAB.

$\frac{dy}{dx} = -2x + 1$, so at x = 1 the gradient is -1

$y - 4 = -1(x - 1)$, so $y = -x + 5$

B: x = 0 gives y = 5. A: y = 0 gives x = 5

Area = ½ × 5 × 5 = $\frac{25}{2}$

Answer: (a) $y = -x + 5$ (b) $\frac{25}{2}$

8 nth term of a cubic sequence

Here are the first five terms of a sequence.
−1, 21, 79, 191, 375
The nth term is of the form $an^3 + bn + c$.
Find the values of a, b and c.

1st differences: 22, 58, 112, 184. 2nd: 36, 54, 72. 3rd: 18, 18
Third differences = 6a = 18 so a = 3
Subtract $3n^3$: −4, −3, −2, −1, 0
This is linear with common difference 1, so b = 1, and c = −4 − 1 = −5
Answer: a = 3, b = 1, c = −5

9 Equal roots and real roots

Show that the equation $x^2 + 3kx - 7 = 0$ has two distinct real solutions for all values of k.

$b^2 - 4ac = (3k)^2 - 4 \times 1 \times (-7)$

$= 9k^2 + 28$

$9k^2$ is never negative, so $9k^2 + 28$ is always at least 28, which is greater than 0

So $b^2 - 4ac > 0$ for all k, and there are always two distinct real solutions

Answer: Shown

10 Growth and decay

A car is worth £5,000 when new.
Its value decreases by 10% each year.
After how many whole years will it first be worth less than £3,750? (Calculator)

Multiplier each year = 0.9

After 2 years: £5,000 × 0.9^2 = £4,050

After 3 years: £5,000 × 0.9^3 = £3,645

So the first time it is less than £3,750 is after 3 years

Answer: 3 years