Day 316 **IGCSE 10-a-day: Model Solutions**

1 Second derivative

$y = -2x^2 + \frac{3}{x^2}$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Work out the value of $\frac{d^2y}{dx^2}$ when $x = 2$.

$\frac{dy}{dx} = -4x - 6x^{-3}$

$\frac{d^2y}{dx^2} = -4 + 18x^{-4}$

$x = 2$: $\frac{d^2y}{dx^2} = -\frac{23}{8}$

Answer: (a) $\frac{dy}{dx} = -4x - \frac{6}{x^3}$, $\frac{d^2y}{dx^2} = -4 + \frac{18}{x^4}$ (b) $-\frac{23}{8}$

2 Averages with algebra

Here are five numbers: $2x - 3$, $2x + 1$, $x + 4$, $3x + 2$, $3x - 2$.
The mean of the five numbers is 18.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = $(2x - 3) + (2x + 1) + (x + 4) + (3x + 2) + (3x - 2) = 18 \times 5 = 90$
$11x + 2 = 90$, so $11x = 88$ and $x = 8$
The numbers are 13, 17, 12, 26, 22
(b) In order: 12, 13, 17, 22, 26, so the median is 17
Answer: (a) x = 8 (b) 17

3 Differentiating polynomials

Find $\frac{dy}{dx}$ when $y = 4x^3 + 3x - 8$

Differentiate each term: $x^n \rightarrow nx^{n-1}$ and a constant $\rightarrow 0$
$4x^3 \rightarrow 12x^2$, $3x \rightarrow 3$
$\frac{dy}{dx} = 12x^2 + 3$

Answer: $\frac{dy}{dx} = 12x^2 + 3$

4 Venn diagram: shaded region notation

Write down the set notation for the region shaded in the Venn diagram.

Identify which regions are shaded, using the circles X, Y and Z
The shaded region is $(Y \cap X \cap Z)'$
Answer: $(Y \cap X \cap Z)'$

5 Similar shapes

Two cones, P and Q, are mathematically similar.
The volume of P is 4 cm^3 and the volume of Q is 32 cm^3.
The curved surface area of P is 2 cm^2.
Work out the curved surface area of Q.

Ratio of volumes = $4 : 32 = 1^3 : 2^3$
So the ratio of lengths is 1 : 2
Ratio of areas = $1^2 : 2^2$, so scale factor = 4
$2 \times 4 = 8$ cm^2
Answer: 8 cm^2

6 Show that dy/dx factorises

$y = (x - 3)^2(x - 4)$

Show that $\frac{dy}{dx} = (x - 3)(3x - 11)$.

Expand: $y = x^3 - 10x^2 + 33x - 36$

$\frac{dy}{dx} = 3x^2 - 20x + 33$

Factorise: $3x^2 - 20x + 33 = (x - 3)(3x - 11)$

Answer: $\mathbf{\frac{dy}{dx} = (x - 3)(3x - 11)}$

7 Trigonometric graphs

The diagram shows the graph of $y = a \sin x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At $x = 90$, $\sin x° = 1$, so $a + c = 5$
At $x = 270$, $\sin x° = -1$, so $-a + c = 9$
Add: $2c = 14$, so $c = 7$
Subtract: $2a = -4$, so $a = -2$
Answer: $a = -2$, $c = 7$

8 Finding a percentage rate

Hana invests £5,000 in an account paying compound interest.
The interest rate is the same percentage each year.
After 3 years the account holds £10,985.
Work out the annual percentage interest rate.

Multiplier for 3 years = £10,985 ÷ £5,000 = 2.197
Yearly multiplier = the cube root of 2.197 = 1.3
30% increase each year
Answer: 30%

9 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 65$
$y = -2x - 15$

Substitute into the first equation: $x^2 + (-2x - 15)^2 = 65$
Expand: $5x^2 + 60x + 160 = 0$
Divide by 5: $x^2 + 12x + 32 = 0$
Factorise: $(x + 8)(x + 4) = 0$, so $x = -8$ or $x = -4$
$y = -2x - 15$ gives $y = 1$ and $y = -7$
Answer: $x = -8$, $y = 1$ and $x = -4$, $y = -7$

10 Finding a matrix from MA = B

$A = \begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -7 \\ -1 & 0 \end{pmatrix}$.
MA = B, where M is a 2 × 2 matrix.
Find M.

$MA = B$ so $M = BA^{-1}$

$A^{-1} = \begin{pmatrix} -3 & -1 \\ -2 & -1 \end{pmatrix}$

$M = \begin{pmatrix} 5 & -7 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -3 & -1 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix}$

Answer: $\mathbf{M = \begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix}}$