Day 315 **IGCSE 10-a-day: Model Solutions**

1 Function values and composites

$f(x) = -3x + 2$ and $g(x) = x^2 + 2x - 1$.
(a) Work out g(1).
(b) Work out fg(−1).

(a) $g(1) = 2$
(b) $g(-1) = -2$, so $fg(-1) = f(-2) = 8$
Answer: (a) 2 (b) 8

2 Tangent at a turning point

The graph of $y = -2x^2 + 8x - 4$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = -4x + 8$

At P, $\frac{dy}{dx} = 0$, so $-4x + 8 = 0$ and $x = 2$

$y = 4$ at P
The gradient is 0, so the tangent is horizontal: $y = 4$
Answer: y = 4

3 Tests of divisibility

The 4-digit number 504k is a multiple of 8.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 8 if its last three digits form a multiple of 8
Last three digits: 04k
Multiples of 8 of this form: 040, 048
Answer: k = 0 or 8

4 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by
$h = -4t^2 + 40t$
(a) Work out the greatest height reached by the ball.
(b) Work out the velocity of the ball when t = 6. State whether it is moving up or down.

$\frac{dh}{dt} = -8t + 40 = 0$ gives $t = 5$

$\frac{d^2h}{dt^2} = -8 < 0$ so maximum. $h = 100$

When t = 6: $\frac{dh}{dt} = 40 - 8 \times 6 = -8$

Answer: (a) 100 m (b) −8 m/s, moving down

5 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 10 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 20 ÷ 10 = 2 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 10 \times 20 + 20 \times 10 + \frac{1}{2} \times 10 \times 20 = 100 + 200 + 100$
$= 400$
Answer: (a) 2 m/s^2 (b) 400 m

6 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 9 cm, DB = 6 cm and the area of triangle ADE is 180 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 9 + 6 = 15 cm

Length scale factor = $15 \div 9 = \frac{5}{3}$

Area scale factor = $(\frac{5}{3})^2 = \frac{25}{9}$

Area of triangle ABC = $180 \times \frac{25}{9} = 500$ cm^2

Trapezium = $500 - 180 = 320$ cm^2

Answer: 320 cm^2

7 Reverse percentages

The price of a watch is increased by 50%.
Later the new price is reduced by 40% in a sale.
The sale price is £297.
Work out the price before the increase. (Calculator)

Multipliers: 1.5 and 0.6
Overall multiplier = $1.5 \times 0.6 = 0.9$
Original × 0.9 = £297
Original = £297 ÷ 0.9 = £330
Answer: £330

8 Linear programming in context

A bakery makes x cakes and y loaves in a morning.
Each cake needs 5 minutes of oven time and 1 kg of flour. Each loaf needs 3 minutes of oven time and 3 kg of flour.
There are 75 minutes of oven time and 39 kg of flour available.
The profit is £3 for each cake and £7 for each loaf.
(a) Write down two inequalities, other than $x \geq 0$, $y \geq 0$. (b) Find the greatest profit.

(a) $5x + 3y \leq 75$ and $x + 3y \leq 39$
(b) Vertices of the feasible region: (0, 0), (0, 13), (9, 10), (15, 0)
Profit = 3x + 7y: 0, 91, 97, 45
Greatest profit = £97 with x = 9, y = 10
Answer: (a) $5x + 3y \leq 75$, $x + 3y \leq 39$ (b) £97

9 Normal to y = kx^2: show that

P is the point $(p, \frac{1}{2}p^2)$ on the curve $y = \frac{1}{2}x^2$, where $p \neq 0$.

The normal to the curve at P meets the y-axis at the point Q.

Show that the y coordinate of Q is $\frac{1}{2}p^2 + 1$.

$\frac{dy}{dx} = x$, so the gradient of the tangent at P is p

Normal gradient = $-\frac{1}{p}$

Normal: $y - \frac{1}{2}p^2 = -\frac{1}{p}(x - p)$

Put x = 0: $y = \frac{1}{2}p^2 + \frac{p}{p} = \frac{1}{2}p^2 + 1$

Answer: shown: $y = \frac{1}{2}p^2 + 1$

10 Nested surds

Show that $\sqrt{17 + 12\sqrt{2}} = 3 + 2\sqrt{2}$

Square the right-hand side: $(3 + 2\sqrt{2})^2 = 9 + 12\sqrt{2} + 8$

$= 17 + 12\sqrt{2}$

So $\sqrt{17 + 12\sqrt{2}} = 3 + 2\sqrt{2}$

Answer: $\sqrt{\mathbf{17 + 12\sqrt{2}}} = \mathbf{3 + 2\sqrt{2}}$