Day 314 **IGCSE 10-a-day: Model Solutions**

1 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where $s = 2t^3 - 6t^2 + 3t + 5$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when t = 3.

$v = \frac{ds}{dt} = 6t^2 - 12t + 3$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 12t - 12$

When t = 3: v = 21 and a = 24

Answer: (a) 21 m/s (b) 24 m/s^2

2 Nested surds

Show that $\sqrt{6 + 2\sqrt{5}} = 1 + \sqrt{5}$

Square the right-hand side: $(1 + \sqrt{5})^2 = 1 + 2\sqrt{5} + 5$

$= 6 + 2\sqrt{5}$

So $\sqrt{6 + 2\sqrt{5}} = 1 + \sqrt{5}$

Answer: $\sqrt{6 + 2\sqrt{5}} = 1 + \sqrt{5}$

3 Successive percentage change

The population of a town is decreased by 30%.

The new amount is then decreased by 50%.

Work out the overall percentage change.

Multipliers: 0.7 and 0.5

$0.7 \times 0.5 = 0.35$

0.35 is a decrease of 65%

Answer: 65% decrease

4 Gradient where the curve crosses the x-axis

A curve has equation $y = (x - 1)(x + 1)(x + 2)$.

Work out the gradient of the curve at each point where it crosses the x-axis.

Expand: $y = x^3 + 2x^2 - x - 2$

$\frac{dy}{dx} = 3x^2 + 4x - 1$

The curve crosses the x-axis at $x = -2, -1$ and 1

Gradients: $x = -2$ gives 3, $x = -1$ gives -2, $x = 1$ gives 6

Answer: 3 at $x = -2$, -2 at $x = -1$, 6 at $x = 1$

5 Trapezium with vectors

OABC is a trapezium. OA is parallel to CB and $CB = \frac{1}{4}OA$.

$\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$.

(a) Find $\mathbf{AB}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

(b) AM : MB = 1 : 1. Find $\mathbf{OM}$.

(a) $\mathbf{CB} = \frac{1}{4}\mathbf{a}$

$\mathbf{OB} = \mathbf{OC} + \mathbf{CB} = \frac{1}{4}\mathbf{a} + \mathbf{c}$

$\mathbf{AB} = -\mathbf{OA} + \mathbf{OB} = -\frac{3}{4}\mathbf{a} + \mathbf{c}$

(b) $\mathbf{OM} = \mathbf{OA} + \frac{1}{2}\mathbf{AB} = \mathbf{a} + \frac{1}{2}(-\frac{3}{4}\mathbf{a} + \mathbf{c})$

$= \frac{5}{8}\mathbf{a} + \frac{1}{2}\mathbf{c}$

Answer: (a) $-\frac{3}{4}\mathbf{a} + \mathbf{c}$ (b) $\frac{5}{8}\mathbf{a} + \frac{1}{2}\mathbf{c}$

6 Show that the gradient is always positive

A curve has equation $y = x^3 + x^2 + 4x + 2$.
Show that y is an increasing function for all values of x.

$\frac{dy}{dx} = 3x^2 + 2x + 4$

Discriminant $= 2^2 - 4 \times 3 \times 4 = -44$

The discriminant is negative, so $\frac{dy}{dx} = 0$ has no real solutions

The coefficient of x^2 is positive, so $\frac{dy}{dx} > 0$ for all x

So y is increasing for all values of x

Answer: Discriminant = −44 < 0, so dy/dx > 0 for all x

7 Transformations of graphs

The graph of y = f(x) has a minimum point at (1, −5).
Write down the coordinates of the turning point of the graph of
(a) y = f(−x)
(b) y = f(x − 3) − 3

(a) Start with the turning point (1, −5)
Apply the transformation to the point: (−1, −5)
(b) f(x − 3) changes the x-coordinate to 4, and f(x) − 3 changes the y-coordinate to −8
Turning point: (4, −8)

Answer: (a) (−1, −5) (b) (4, −8)

8 Three sets from information

63 students were asked which activities they do.
A: study Art B: study Music C: study Drama
n(A) = 34, n(B) = 31, n(C) = 24, n(A ∩ B) = 11, n(A ∩ C) = 14, n(B ∩ C) = 12, n(A ∩ B ∩ C) = 5.
(a) Work out how many are in none of A, B and C.
(b) Work out how many are in exactly one of A, B and C.

n(A ∪ B ∪ C) = 34 + 31 + 24 − 11 − 14 − 12 + 5 = 57
(a) Number in none of the sets = 63 − 57 = 6
(b) Only A = 34 − 11 − 14 + 5 = 14
Only B = 13, only C = 3, so exactly one = 14 + 13 + 3 = 30

Answer: (a) 6 (b) 30

9 Trigonometric graphs

The diagram shows the graph of y = a cos bx, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when x = 210°.

(a) The maximum value is 3 and the minimum is −3, so a = 3
The graph completes 2 full cycles between 0° and 360°, so b = 2
(b) y = 3 cos (2 × 210°) = 3 cos 420°

cos 60° = $\frac{1}{2}$, so y = $\frac{3}{2}$

Answer: (a) a = 3, b = 2 (b) y = $\frac{3}{2}$

10 Tangent to a circle: proof

Show that the line $y = x + 4$ is a tangent to the circle $x^2 + y^2 = 8$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = x + 4$ into $x^2 + y^2 = 8$: $x^2 + (x + 4)^2 = 8$

Expand: $2x^2 + 8x + 8 = 0$

$b^2 - 4ac = 8^2 - 4 \times 2 \times 8 = 64 - 64 = 0$

$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent

$x = \frac{-8}{2 \times 2} = -2$, and $y = (-2) + 4 = 2$. The point is $(-2, 2)$

Answer: $(-2, 2)$