Day 313 **IGCSE 10-a-day: Model Solutions**

1 Where a line meets the axes

The line L has equation $4x + 3y = 12$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the length of AB.

(a) At A, $y = 0$: $4x = 12$, so $x = 3$. A is (3, 0)
At B, $x = 0$: $3y = 12$, so $y = 4$. B is (0, 4)
(b) $AB^2 = 3^2 + 4^2 = 25$
$AB = \sqrt{25} = 5$
Answer: (a) A(3, 0), B(0, 4) (b) 5

2 Stationary points of a cubic

A curve has equation $y = -x^3 + 3x^2 + 9x - 7$.
Find the coordinates of the stationary points of the curve.

$\frac{dy}{dx} = -3x^2 + 6x + 9$
Stationary points: $\frac{dy}{dx} = 0$, so $-3(x + 1)(x - 3) = 0$
$x = -1$ or $x = 3$
$y = -12$ when $x = -1$ and $y = 20$ when $x = 3$
Answer: (−1, −12) and (3, 20)

3 Combined and changing means

The mean of 7 numbers is 68.
One of the numbers, 86, is removed.
Work out the mean of the remaining 6 numbers.

Total of 7 numbers = $7 \times 68 = 476$
Total after removing 86 = $476 - 86 = 390$
New mean = $390 \div 6 = 65$
Answer: 65

4 Increasing and decreasing intervals

$y = 2x^3 - 3x^2 + 4$
Find the range of values of x for which y is increasing.

$\frac{dy}{dx} = 6x^2 - 6x = 6x(x - 1)$
$\frac{dy}{dx} = 0$ when $x = 0$ and $x = 1$
The graph of $\frac{dy}{dx}$ against x is a U-shaped quadratic
y is increasing when $\frac{dy}{dx} > 0$ and decreasing when $\frac{dy}{dx} < 0$
So y is increasing when $x < 0$ or $x > 1$
Answer: $x < 0$ or $x > 1$

5 Combining means

The mean of the heights in cm for the Year 10 students is 75.
The mean of the heights in cm for the Year 11 students is 60.
The mean for all of the Year 10 students and Year 11 students together is 70.
Work out the ratio of the number of Year 10 students to the number of Year 11 students.

Let there be p Year 10 students and q Year 11 students
Total = $75p + 60q$, and the overall mean gives $75p + 60q = 70(p + q)$
$5p = 10q$
$p : q = 10 : 5 = 2 : 1$
Answer: 2 : 1

6 Show that dy/dx factorises

$y = (x - 2)^2(x - 3)$

Show that $\frac{dy}{dx} = (x - 2)(3x - 8)$.

Expand: $y = x^3 - 7x^2 + 16x - 12$

$\frac{dy}{dx} = 3x^2 - 14x + 16$

Factorise: $3x^2 - 14x + 16 = (x - 2)(3x - 8)$

Answer: $\frac{dy}{dx} = (x - 2)(3x - 8)$

7 3D trigonometry: pyramid

VABCD is a pyramid. The base ABCD is a square with sides of 5 cm.
The vertex V is vertically above the centre O of the base. VA = 15 cm.
(a) Work out the vertical height VO.
(b) Work out the angle between VA and the base. (Calculator)

(a) AC = $5\sqrt{2}$ so AO = 3.536 cm

$VO^2 = 15^2 - 3.536^2 = 212.5$

VO = 14.6 cm

(b) $\cos VAO = \frac{AO}{VA} = \frac{3.536}{15}$, angle = 76.4°

Answer: (a) 14.6 cm (b) 76.4°

8 Angles of elevation

A point A is on level ground in line with the base of a cliff. (Calculator)
The angle of elevation of the top of the cliff from A is 39°.
B is a point on the same line, 100 m further from the cliff than A.
The angle of elevation of the top of the cliff from B is 25°.
Work out the height of the cliff. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 39° = \frac{h}{x}$, so $x = h \div \tan 39°$

From B: $\tan 25° = \frac{h}{x + 100}$, so $x + 100 = h \div \tan 25°$

Subtract: $100 = h \div \tan 25° - h \div \tan 39° = h(2.1445 - 1.2349)$

h = 100 ÷ 0.90961 = 109.9 m (1 d.p.)

Answer: 109.9 m

9 Ratio and mixtures

Container A holds juice and water in the ratio 6 : 4.
Container B holds juice and water in the ratio 3 : 1.
20 litres from container A is mixed with some of container B.
The ratio of juice to water in the mixture is 2 : 1.
Work out how many litres were taken from container B.

Let y litres be taken from B

juice: $12 + \frac{3y}{4}$

water: $8 + \frac{1y}{4}$

Ratio 2 : 1, so $1(12 + \frac{3y}{4}) = 2(8 + \frac{1y}{4})$

$(\frac{1}{4})y = 4$, so y = 16

Answer: 16 litres

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10 Laws of indices

Write $\frac{16^n \times 8^{n-1}}{4^n}$ in the form 2^{an+c}

where a and c are integers.

$16 = 2^4$, $8 = 2^3$, $4 = 2^2$

$16^n = 2^{4(n)}$, $8^{n-1} = 2^{3(n-1)}$, $4^n = 2^{2n}$

Exponent: $4(n) + 3(n - 1) - 2n = 5n - 3$

Answer: a = 5, c = −3