Day 312 **IGCSE 10-a-day: Model Solutions**

1 Largest domain and range

$f(x) = \frac{6}{x+1}$.

(a) State the largest possible domain of f.

(b) State the range of f for this domain.

(a) The denominator cannot be 0, so $x + 1 \neq 0$

Domain: $x \neq -1$

(b) A fraction with a non-zero numerator can never equal 0

Answer: (a) $x \neq -1$ (b) $f(x) \neq 0$

2 Square roots as powers

$y = -8\sqrt{x} - \frac{8}{\sqrt{x}}$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = -8x^{1/2} - 8x^{-1/2}$

$-8x^{1/2} \rightarrow -4x^{-1/2}$, $-8x^{-1/2} \rightarrow 4x^{-3/2}$

$\frac{dy}{dx} = -4x^{-1/2} + 4x^{-3/2}$

Answer: $\frac{dy}{dx} = -4x^{-1/2} + 4x^{-3/2}$

3 Equations with fractions

Solve $\frac{x+3}{3} - \frac{x-3}{5} = 2$

Multiply every term by 15: $5(x + 3) - 3(x - 3) = 30$

Expand: $2x + 24 = 30$

$2x = 6$

$x = 3$

Answer: $x = 3$

4 Rationalising the denominator

Rationalise the denominator of $\frac{11}{4\sqrt{6}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{6}$:

$\frac{11}{4\sqrt{6}} = \frac{11\sqrt{6}}{24}$

Answer: $\frac{11\sqrt{6}}{24}$

5 Recasting solids

A solid metal sphere of radius 6 cm is melted down and recast into a solid cylinder of radius 3 cm.
Work out the height of the cylinder.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of sphere = $\frac{4}{3} \times \pi \times 6^3 = 288\pi$

Volume of cylinder = $\pi \times 3^2 \times h = 9\pi h$

$9h = 288$

$h = 32$ cm

Answer: 32 cm

6 Compound interest

Lena invests £6,000.
The account pays compound interest of 6% a year for the first 3 years,
then 2.5% a year for the next 3 years.
Work out the total amount after 6 years, to the nearest penny. (Calculator)

Multipliers: 1.06 for 3 years, then 1.025 for 3 years
£6,000 × 1.06^3 × 1.025^3
= £7,695.56
Answer: £7,695.56

7 Ratio and algebra

In a coach, the ratio of the number of adults to the number of children is 7 : 8.
Then 2 adults are removed and 8 more children are added.
The ratio is now 4 : 5.
How many adults were there at the start?

Let the numbers at the start be 7k and 8k
(7k − 2) : (8k + 8) = 4 : 5, so 5(7k − 2) = 4(8k + 8)
35k − 10 = 32k + 32
3k = 42, so k = 14
Number of adults at the start: 7 × 14 = 98
Answer: 98

8 Stationary point of a quartic

$y = x^4 + 4x$
Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 4x^3 + 4$
$\frac{dy}{dx} = 0$: $4x^3 + 4 = 0$, so $x^3 = -1$
$x = -1$ and $y = -3$
$\frac{d^2y}{dx^2} = 12x^2$
At $x = -1$: $\frac{d^2y}{dx^2} = 12$, so a minimum
Answer: (−1, −3), minimum

9 Quadrilateral: cosine rule, then area

In quadrilateral ABCD, AB = 11 cm, BC = 6 cm, CD = 12 cm,
angle ABC = 89° and angle ACD = 79°.
Work out the area of ABCD. Give your answer correct to 3 significant figures. (Calculator)

$AC^2 = 11^2 + 6^2 - 2 \times 11 \times 6 \times \cos 89°$ so AC = 12.438 cm
Area ABC = ½ × 11 × 6 × sin 89° = 32.99
Area ACD = ½ × 12.438 × 12 × sin 79° = 73.26
Total area = 32.99 + 73.26 = 106
Answer: 106 cm^2

10 Tangent meets the curve again

A curve has equation $y = x^3 + 4x^2 + 6x + 3$.
The tangent to the curve at the point A, where $x = -3$, meets the curve again at the point B.
Find the coordinates of B.

$\frac{dy}{dx} = 3x^2 + 8x + 6$, so the gradient at A is 9, and A = $(-3, -6)$

Tangent: $y = 9x + 21$

Equate: $x^3 + 4x^2 - 3x - 18 = 0$, which has a repeated root $x = -3$

Sum of the roots = -4, so $-3 + (-3) + x = -4$ and $x = 2$

$y = 39$

Answer: B (2, 39)