Day 311 **IGCSE 10-a-day: Model Solutions**

1 Divide then differentiate

$y = \frac{x^4 - 7x^3 - x}{x}$

Find $\frac{dy}{dx}$.

Divide each term by x: $y = x^3 - 7x^2 - 1$

$x^3 \to 3x^2$, $-7x^2 \to -14x$

$\frac{dy}{dx} = 3x^2 - 14x$

Answer: $\mathbf{\frac{dy}{dx} = 3x^2 - 14x}$

2 Joint and inverse variation

y is directly proportional to the square of x and inversely proportional to the square of z.
When x = 3 and z = 2, y = 18.
Work out the value of y when x = 1 and z = 2.

$y = \frac{kx^2}{z^2}$

$18 = \frac{k \times 9}{4}$, so $k = 18 \times 4 \div 9 = 8$

$y = \frac{8 \times 1}{4} = 2$

Answer: y = 2

3 Coordinates and gradient at the y-intercept

$y = 2x^3 - x^2 + 4x - 8$
The curve crosses the y-axis at the point A.
Work out the coordinates of A and the gradient of the curve at A.

When x = 0, y = −8, so A is (0, −8)

$\frac{dy}{dx} = 6x^2 - 2x + 4$

When x = 0, $\frac{dy}{dx} = 4$

Answer: A = (0, −8), gradient 4

4 Average speed

Fatima drives 105 km at an average speed of 70 km/h.
The drive then continues for another 15 km at an average speed of 30 km/h.
Work out the average speed for the whole journey.

Time for first part = 105 ÷ 70 = 1.5 hours
Time for second part = 15 ÷ 30 = 0.5 hours
Total distance = 120 km, total time = 2 hours
Average speed = 120 ÷ 2 = 60 km/h
Answer: 60 km/h

5 Venn diagram: reading set notation

The Venn diagram shows information about 14 employees.
A: speak German B: speak Italian
(a) Find $n(A' \cup B')$.
(b) Find $n(A' \cap B)$.

(a) $n(A' \cup B') = 2 + 5 + 4 = 11$
(b) $n(A' \cap B) = 5$
Answer: (a) 11 (b) 5

6 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 80° and angle CAQ = 62°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 62° (alternate segment theorem)
(b) Angle ACB = angle PAB = 80° (alternate segment theorem)
Answer: (a) 62° (b) 80°

7 Vectors in a triangle with a diagram

In triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$.
P is on AB so that AP : PB = 2 : 1.
(a) Find $\mathbf{AB}$.
(b) Find $\mathbf{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(a) $\mathbf{AB} = -\mathbf{a} + \mathbf{b} = -\mathbf{a} + \mathbf{b}$

(b) $\mathbf{AP} = \frac{2}{3}\mathbf{AB} = \frac{2}{3}(-\mathbf{a} + \mathbf{b})$

$\mathbf{OP} = \mathbf{OA} + \mathbf{AP} = \mathbf{a} + \frac{2}{3}(-\mathbf{a} + \mathbf{b})$

$= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

Answer: (a) $-\mathbf{a} + \mathbf{b}$ (b) $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

8 Percentage change

The value of a car is decreased by 80%.
By what percentage must the new value be increased to get back to the original value?

Let the original be 100. After a 80% decrease it is 20.
To return to 100, increase by 80

Percentage increase = $\frac{80}{20} \times 100 = 400\%$

Answer: 400%

9 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are (x + 3) cm, (x + 5) cm and (x + 7) cm.
The longest side is (x + 7) cm.
(a) Show that $x^2 + 2x - 15 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x + 3)^2 + (x + 5)^2 = (x + 7)^2$
Expand: $x^2 + 6x + 9 + x^2 + 10x + 25 = x^2 + 14x + 49$
Rearrange: $x^2 + 2x - 15 = 0$
(b) Factorise: $(x - 3)(x + 5) = 0$, so $x = 3$ (the other root, $x = -5$, does not give valid side lengths)
The shorter sides are 6 cm and 8 cm, so area = $\frac{1}{2} \times 6 \times 8 = 24$ cm^2

Answer: (b) 24 cm^2

10 Second derivative and nature at a given point

$y = -x^3 + 3x^2 + 9x + 9$

Use the second derivative to show that the curve has a maximum at $x = 3$.

$\frac{dy}{dx} = -3x^2 + 6x + 9$

When $x = 3$: $\frac{dy}{dx} = 0$, so it is a stationary point

$\frac{d^2y}{dx^2} = -6x + 6$

When $x = 3$: $\frac{d^2y}{dx^2} = -12$, which is negative

So there is a maximum at $x = 3$

Answer: maximum at x = 3