Day 310 **IGCSE 10-a-day: Model Solutions**

1 Inverse and repeated functions

f(x) = −4x + 4.

(a) Find $f^{-1}(-8)$.

(b) Express ff(x) in the form px + q.

(a) Solve −4x + 4 = −8: −4x = −12

x = 3 so $f^{-1}(-8) = 3$

(b) ff(x) = −4(−4x + 4) + 4 = 16x − 12

Answer: (a) 3 (b) ff(x) = 16x − 12

2 Gradient function from a given point

A curve has equation $y = 2x^3 + 9x^2 + 12x - 6$.

The point A (−3, k) lies on the curve. Work out the value of k and the gradient of the curve at A.

Substitute x = −3 into the equation of the curve

y = −15 when x = −3, so k = −15

$\frac{dy}{dx} = 6x^2 + 18x + 12$

At A: gradient = 12

Answer: k = −15, gradient = 12

3 Transformations of graphs

The point P(10, 2) lies on the graph of y = f(x).

Write down the coordinates of the image of P on the graph of

(a) $y = \frac{1}{2}f(x)$ (b) y = f(−x)

(a) $y = \frac{1}{2}f(x)$ is a vertical stretch with scale factor $\frac{1}{2}$, so the y coordinate is halved: (10, 1)

(b) y = f(−x) is a reflection in the y-axis, so the x coordinate changes sign: (−10, 2)

Answer: (a) (10, 1) (b) (−10, 2)

4 Inverse proportion in context

12 painters can paint a hall in 30 days.

After 3 days, 3 of the painters leave. The rest carry on working at the same rate.

How many more days does it take to finish?

Total work = 12 × 30 = 360 painter-days

Work done = 12 × 3 = 36, so work left = 324

Workers left = 12 − 3 = 9

Time = 324 ÷ 9 = 36 days

Answer: 36 days

5 Sine rule

The diagram shows triangle ABC. (Calculator)

Work out the length of AC. Give your answer correct to 1 decimal place.

The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 79°} = \frac{6}{\sin 33°}$

$x = \frac{6 \times \sin 79°}{\sin 33°} = 10.814$

x = 10.8 cm (1 d.p.)

Answer: AC = 10.8 cm

6 Differentiating brackets with square roots

$y = \sqrt{x}(x - 4)$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Expand: $y = x^{3/2} - 4x^{1/2}$

$x^{3/2} \to \frac{3}{2}x^{1/2}$, $-4x^{1/2} \to -2x^{-1/2}$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2} - 2x^{-1/2}$

Answer: $\mathbf{\frac{dy}{dx} = \frac{3}{2}x^{1/2} - 2x^{-1/2}}$

7 Solving trigonometric equations

Solve $\cos 2x = -0.5$ for $0 \leq x \leq 180$.

Give your answers correct to 1 decimal place where necessary. (Calculator)

Let $u = 2x$, so $\cos u = -0.5$ with $0 \leq u \leq 360$

Principal value $u = 120.0°$

All values of u: 120.0, 240.0

Divide by 2: x = 60, 120

Answer: x = 60, 120

8 Bounds of speed, density and pressure

A train travels a distance of 560 km, correct to the nearest 10.

The journey takes 70 hours, correct to the nearest whole number.

Work out the lower bound of the speed.

Give your answer correct to 3 significant figures. (Calculator)

$555 \leq$ distance < 565; $69.5 \leq$ time < 70.5

Lower bound of speed = smallest distance ÷ biggest time = 555 ÷ 70.5

= 7.87234... = 7.87 (3 s.f.)

Answer: 7.87 km/h

9 Where two tangents meet

The curve C has equation $y = x^2 - 4x + 6$.

The points A and B on C have x coordinates -2 and 0.

The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = 2x - 4$. At A: gradient -8, A = $(-2, 18)$

Tangent at A: $y = -8x + 2$. At B: gradient -4, B = (0, 6)

Tangent at B: $y = -4x + 6$

Equate: $-8x + 2 = -4x + 6$, so $x = -1$ and $y = 10$

Answer: T $(-1, 10)$

10 Exact trigonometry in triangles

ABC is a right-angled triangle.

Work out the exact value of x. Give your answer in its simplest form.

The diagram is not drawn accurately.

x is the opposite side BC and the known side is the adjacent side AB, so use tan 30°

$\tan 30° = \frac{\sqrt{3}}{3}$

$x = 10\sqrt{3} \times \tan 30° = 10\sqrt{3} \times \frac{\sqrt{3}}{3}$

$x = 10$

Answer: x = 10