Day 31 **IGCSE 10-a-day: Model Solutions**

1 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by

$h = -2t^2 + 8t$

(a) Work out the greatest height reached by the ball.

(b) Work out the velocity of the ball when t = 7. State whether it is moving up or down.

$\frac{dh}{dt} = -4t + 8 = 0$ gives $t = 2$

$\frac{d^2h}{dt^2} = -4 < 0$ so maximum. $h = 8$

When $t = 7$: $\frac{dh}{dt} = 8 - 4 \times 7 = -20$

Answer: (a) 8 m (b) −20 m/s, moving down

2 Percentages

P is 40% less than D.

D is 15% less than S.

Work out P as a percentage of S.

$P = 0.6 \times D$ and $D = 0.85 \times S$

$P = 0.6 \times 0.85 \times S = 0.51 \times S$

$0.51 \times 100 = 51\%$

Answer: 51%

3 Inequalities

(a) Solve $7 - 4x > 7$

(b) n is an integer. Write down all the values of n that satisfy

$-5 \leq 2n - 3 < 1$

(a) Subtract 7: $-4x > 0$

Divide by −4 and reverse the inequality sign: $x < 0$

(b) Add 3: $-2 \leq 2n < 4$

Divide by 2: $-1 \leq n < 2$

Integers: −1, 0, 1

Answer: (a) $x < 0$ (b) −1, 0, 1

4 Speed-time graphs

The graph shows the speed of a car during 16 seconds.

(a) Work out the acceleration during the first 8 seconds.

(b) Work out the average speed over the whole 16 seconds.

(a) Acceleration = change in speed ÷ time = $(16 - 0) \div 8 = 2$ m/s^2

(b) Distance = area under the graph

Trapezium: $\frac{0 + 16}{2} \times 8 = 64$; rectangle: $16 \times 6 = 96$; triangle: $\frac{1}{2} \times 2 \times 16 = 16$

Total = 176 m, so average speed = $176 \div 16 = 11$ m/s

Answer: (a) 2 m/s^2 (b) 11 m/s

5 Tangent to a curve with negative or fractional powers

A curve has equation $y = 6x^{3/2} - 6\sqrt{x}$.

Find the equation of the tangent to the curve at the point where x = 9.

$\frac{dy}{dx} = 9\sqrt{x} - 3x^{-1/2}$

At $x = 9$: $y = 144$ and gradient = 26

$y - 144 = 26(x - 9)$

$y = 26x - 90$

Answer: $y = 26x - 90$

6 Probabilities with an unknown

A biased wheel stops on one of four outcomes with these probabilities.
A: 2x; B: 0.1; C: x + 0.2; D: 0.55
(a) Work out the value of x.
(b) The biased wheel is used 80 times. Work out an estimate for the number of times it stops on A.

(a) Probabilities add up to 1: 2x + 0.1 + x + 0.2 + 0.55 = 1
3x + 0.85 = 1, so 3x = 0.15
x = 0.05
(b) P(A) = 2 × 0.05 = 0.1, so 80 × 0.1 = 8
Answer: (a) x = 0.05 (b) 8

7 Bearings with the sine and cosine rules

A ship leaves a port P and travels 89 km on a bearing of 015° to Q. (Calculator)
It then travels 108 km on a bearing of 325° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 015° + 180° = 195°
Angle PQR = 130° (the difference between the bearings 195° and 325°)
$PR^2 = 89^2 + 108^2 - 2 \times 89 \times 108 \times \cos 130° = 31942$, so PR = 178.7 km
(b) $\cos(QPR) = \frac{89^2 + PR^2 - 108^2}{2 \times 89 \times PR}$, so angle QPR = 27.6°
R is anticlockwise of the direction PQ, so the bearing is 015° − 28° = 347°
Answer: (a) 178.7 km (b) 347°

8 Venn diagram: reading set notation

The Venn diagram shows information about 24 families.
A: have a garden B: have a car
(a) Find $n(A \cup B)'$.
(b) Find $n(B')$.

(a) $n(A \cup B)' = 6$
(b) $n(B') = 5 + 6 = 11$
Answer: (a) 6 (b) 11

9 No stationary points

A curve has equation $y = x^3 + 9x + 3$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 3x^2 + 9 = 3x^2 + 9$

A squared term is never negative, so $\frac{dy}{dx} \geq 9 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 9 (when x = 0)
Answer: (b) 9

10 Transformed curve from a diagram

The diagram shows the graph of y = f(x) with a minimum point at (3, 3).
The dashed curve C is the image of y = f(x) after a single transformation.
The maximum point of C is (3, −3).
Write down the equation of C in terms of f.

The minimum point (3, 3) has moved to (3, −3)
The x-coordinate is unchanged and the y-coordinate has changed sign
This is a reflection in the x-axis
Answer: y = −f(x)