Day 309 **IGCSE 10-a-day: Model Solutions**

1 Expand then differentiate

$y = (x + 6)(x - 4)$

Find $\frac{dy}{dx}$.

Expand: $y = x^2 + 2x - 24$

$x^2 \to 2x$, $2x \to 2$

$\frac{dy}{dx} = 2x + 2$

Answer: $\mathbf{\frac{dy}{dx} = 2x + 2}$

2 Factorising fully

Factorise fully $75x^2 - 12y^2$

Take out the common factor 3: $3(25x^2 - 4y^2)$

$25x^2 - 4y^2$ is a difference of two squares: $(5x)^2 - (2y)^2$

$= (5x - 2y)(5x + 2y)$

Answer: $\mathbf{3(5x - 2y)(5x + 2y)}$

3 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 25°.

(a) Work out the size of angle ACB.

(b) Work out the size of angle COB.

The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°

(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 25°

Angle COB is an exterior angle of triangle OAC: 25 + 25 = 50°

(or: the angle at the centre is twice the angle at the circumference)

Answer: (a) 90° (b) 50°

4 Magnitude of a vector

$\mathbf{v} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

Find $|\mathbf{v}|$. Give your answer as a surd in its simplest form.

$|\mathbf{v}| = \sqrt{(1)^2 + (-4)^2} = \sqrt{1 + 16}$

$= \sqrt{17}$

Answer: $\mathbf{\sqrt{17}}$

5 Rational and irrational numbers

Here are six numbers.

$\sqrt{8} \times \sqrt{2}$, $\sqrt{2} + \sqrt{8}$, $\sqrt{49}$, $0.\dot{4}$, $2\pi - \pi$, $\pi \div 2$

Write down the rational numbers.

Simplify each number:

$\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$; $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$; $\sqrt{49} = 7$

$0.\dot{4} = \frac{4}{9}$; $2\pi - \pi = \pi$; $\pi \div 2$ is irrational

Answer: $\mathbf{\sqrt{8} \times \sqrt{2}, \sqrt{49}, 0.\dot{4}}$

6 Tree diagrams without replacement

A bag contains 6 red beads, 4 blue beads and 1 green bead.
Two beads are taken at random without replacement.
Work out the probability that (a) exactly one is red, (b) they are different colours.

(a) P(exactly one red) = $\frac{6}{11}\times\frac{5}{10} + \frac{5}{11}\times\frac{6}{10} = \frac{6}{11}$

(b) P(different) = 1 − P(same) = $1 - \frac{21}{55} = \frac{34}{55}$

Answer: (a) $\frac{6}{11}$ (b) $\frac{34}{55}$

7 Points with a given gradient on y = ax + b/x

A curve has equation $y = 3x - \frac{4}{x}$.

Find the coordinates of the points on the curve where the gradient is 4.

$\frac{dy}{dx} = 3 + \frac{4}{x^2}$

Set $\frac{dy}{dx} = 4$: $-1 = \frac{-4}{x^2}$

$x^2 = 4$, so $x = 2$ or $x = -2$

$y = 4$ when $x = 2$ and $y = -4$ when $x = -2$

Answer: (2, 4) and (−2, −4)

8 Show that: consecutive terms of a quadratic sequence

The nth term of a sequence is $4n^2 + 5n - 4$.

(a) Show that the difference between the (n + 1)th term and the nth term is 8n + 9.

(b) Two consecutive terms of the sequence differ by 49. Find the value of n for the first of these two terms.

(a) (n + 1)th term $= 4(n+1)^2 + 5(n+1) - 4 = 4n^2 + 13n + 5$

Subtract the nth term: $4n^2 + 13n + 5 - (4n^2 + 5n - 4) = 8n + 9$

(b) $8n + 9 = 49$ so $8n = 40$

$n = 5$

Answer: (b) n = 5

9 Open box of fixed surface area

An open box has a square base of side x cm and height h cm. The total outside surface area is 300 cm^2.

(a) Show that the volume of the box is $V = \frac{1}{4}(300x - x^3)$ cm^3.

(b) Find the maximum volume of the box, proving that it is a maximum.

$x^2 + 4xh = 300$, so $h = \frac{300 - x^2}{4x}$

$V = x^2h = \frac{x(300 - x^2)}{4} = \frac{1}{4}(300x - x^3)$

$\frac{dV}{dx} = -\frac{3}{4}x^2 + 75 = 0$, so $x^2 = 100$ and $x = 10$

$\frac{d^2V}{dx^2} = -\frac{3}{2}x = -15 < 0$, so a maximum

$V = 500$

Answer: 500 cm^3

10 Joint and inverse variation

y is directly proportional to x and inversely proportional to z.
When x = 3 and z = 6, y = 1.
Work out the value of y when x = 6 and z = 6.

$y = \frac{kx}{z}$

$1 = \frac{k \times 3}{6}$, so $k = 1 \times 6 \div 3 = 2$

$y = \frac{2 \times 6}{6} = 2$

Answer: y = 2