Day 308 **IGCSE 10-a-day: Model Solutions**

1 Parallel vectors

$\mathbf{a} = \begin{pmatrix} k \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

$\mathbf{a}$ is parallel to $\mathbf{b}$.

Find the value of k.

Parallel vectors are multiples of each other: $\mathbf{a} = \lambda\mathbf{b}$

$4 = -4\lambda$ gives $\lambda = -1$

$k = -1 \times 3 = -3$

Answer: $k = -3$

2 Fractions

Work out $1\frac{2}{3} \times 2\frac{2}{3} - 3\frac{1}{3}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{5}{3} \times \frac{8}{3} = \frac{40}{9}$

$\frac{40}{9} - \frac{10}{3} = \frac{10}{9}$

$= 1\frac{1}{9}$

Answer: $1\frac{1}{9}$

3 Gradient with fractional or negative powers

The equation of a curve is $y = x^2 + \frac{7}{x}$.

Work out the gradient of the curve at the point where x = 2.

$\frac{dy}{dx} = 2x - 7x^{-2}$

When x = 2: gradient = $\frac{9}{4}$

Answer: $\frac{9}{4}$

4 Inverse proportion in context

15 workers can build a wall in 10 days.

After 6 days, 3 of the workers leave. The rest carry on working at the same rate.

How many more days does it take to finish?

Total work = 15 × 10 = 150 worker-days

Work done = 15 × 6 = 90, so work left = 60

Workers left = 15 − 3 = 12

Time = 60 ÷ 12 = 5 days

Answer: 5 days

5 Quadratic sequences

Here are the first five terms of a quadratic sequence.

0, 11, 30, 57, 92

Find an expression for the nth term.

First differences: 11, 19, 27, 35

Second differences: 8, 8, 8, so $a = 8 \div 2 = 4$

Subtract $4n^2$ from each term: −4, −5, −6, −7, −8

The remaining linear sequence has nth term $-n - 3$

nth term = $4n^2 - n - 3$

Answer: $4n^2 - n - 3$

6 Radius from chord and height of arc

A circular arc has a chord AB of length 24 cm.
The greatest distance between the chord and the arc is 6 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = $r - 6$

AM = $24 \div 2 = 12$. Pythagoras in triangle OMA: $r^2 = (r - 6)^2 + 12^2$

$r^2 = r^2 - 12r + 36 + 144$

$12r = 180$, so $r = 15$ cm

Answer: 15 cm

7 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = 2t^3 - 24t^2 + 72t + 3$ $(t \geq 0)$
Find the acceleration of P at each instant when it is at rest.

$v = \frac{ds}{dt} = 6t^2 - 48t + 72$

P is at rest when $v = 0$: $6t^2 - 48t + 72 = 0$

$6(t - 2)(t - 6) = 0$, so $t = 2$ or $t = 6$

$a = \frac{d^2s}{dt^2} = 12t - 48$

$t = 2$: $a = -24$. $t = 6$: $a = 24$

Answer: -24 m/s^2 when t = 2 and 24 m/s^2 when t = 6

8 Angles of elevation

A point A is on level ground in line with the base of a mast. (Calculator)
The angle of elevation of the top of the mast from A is 46°.
B is a point on the same line, 50 m further from the mast than A.
The angle of elevation of the top of the mast from B is 33°.
Work out the height of the mast. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 46° = \frac{h}{x}$, so $x = h \div \tan 46°$

From B: $\tan 33° = \frac{h}{x + 50}$, so $x + 50 = h \div \tan 33°$

Subtract: $50 = h \div \tan 33° - h \div \tan 46° = h(1.5399 - 0.96569)$

$h = 50 \div 0.57418 = 87.1$ m (1 d.p.)

Answer: 87.1 m

9 Second derivative and nature at a given point

$y = -x^3 - 3x^2 - 2$
Use the second derivative to show that the curve has a minimum at $x = -2$.

$\frac{dy}{dx} = -3x^2 - 6x$

When $x = -2$: $\frac{dy}{dx} = 0$, so it is a stationary point

$\frac{d^2y}{dx^2} = -6x - 6$

When $x = -2$: $\frac{d^2y}{dx^2} = 6$, which is positive

So there is a minimum at $x = -2$

Answer: minimum at x = −2

10 Exact values from a given ratio

θ is an acute angle and $\tan\theta = \frac{1}{8}$.

Find the exact value of (a) $\sin\theta$ (b) $\cos\theta$.

Give each answer as a fraction, rationalising any denominator.

Right-angled triangle: opposite = 1, adjacent = 8

Hypotenuse = $\sqrt{1^2 + 8^2} = \sqrt{65}$

(a) $\sin\theta = \frac{1}{\sqrt{65}} = \frac{\sqrt{65}}{65}$

(b) $\cos\theta = \frac{8}{\sqrt{65}} = \frac{8\sqrt{65}}{65}$

Answer: (a) $\frac{\sqrt{65}}{65}$ (b) $\frac{8\sqrt{65}}{65}$