Day 306 **IGCSE 10-a-day: Model Solutions**

1 Tangent where the curve crosses the y-axis

The curve $y = -2x^2 - x - 4$ crosses the y-axis at the point A.
The tangent to the curve at A meets the x-axis at B.
Find the coordinates of B.

At A, $x = 0$ and $y = -4$
$\frac{dy}{dx} = -4x - 1$, so the gradient at A is -1
Tangent: $y = -x - 4$
At B, $y = 0$: $-x - 4 = 0$, so $x = -4$
Answer: $(-4, 0)$

2 Time zones

A flight leaves London at 22:15 (London time).
It takes 12 hours 30 minutes.
When it is 12:00 in London it is 21:00 in Sydney.
At what time, in Sydney time, does the flight land? Say if it is on a different day.

Arrival in London time = 22:15 + 12 h 30 min = 10:45 the next day
Sydney is 9 hours ahead of London, so add 9 hours
19:45 the next day
Answer: 19:45 the next day

3 f′(x) notation

$f(x) = -3x^3 - 4x^2 - 4$
(a) Find $f'(x)$.
(b) Work out the value of $f'(-2)$.

(a) $-3x^3 \to -9x^2$, $-4x^2 \to -8x$
$f'(x) = -9x^2 - 8x$
(b) $f'(-2) = -20$
Answer: (a) $f'(x) = -9x^2 - 8x$ (b) -20

4 Solving trigonometric equations

Solve $\tan x = -\sqrt{3}$ for $0° \le x \le 360°$.
Do not use a calculator.

The related acute angle satisfies $\tan \theta = \sqrt{3}$, so $\theta = 60°$
tangent is negative in the 2nd and 4th quadrants, so $x = 180° - 60°$ and $x = 360° - 60°$
$x = 120°$ and $x = 300°$
Answer: $x = 120°$ or $x = 300°$

5 Density of an alloy

An alloy contains 146 g of tin and 178 g of nickel.
The density of tin is 7.3 g/cm^3. The density of nickel is 8.9 g/cm^3.
Work out the density of the alloy.

Volume of tin = $146 \div 7.3 = 20$ cm^3
Volume of nickel = $178 \div 8.9 = 20$ cm^3
Total mass = 324 g, total volume = 40 cm^3
Density = $324 \div 40 = 8.1$ g/cm^3
Answer: 8.1 g/cm^3

6 Composite functions

$f(x) = 2x - 7$ and $g(x) = \frac{12}{x}$

(a) Find $gf(x)$.

(b) Solve $gf(x) = \frac{12}{5}$

(a) $gf(x) = g(2x - 7) = \frac{12}{2x - 7}$

(b) $\frac{12}{2x - 7} = \frac{12}{5}$, so $2x - 7 = 12 \div \frac{12}{5} = 5$

$2x = 12$

$x = 6$

Answer: (a) $\frac{12}{2x - 7}$ (b) $x = 6$

7 Bounds of a compound expression

$p = 11$ and $r = 6.2$, each correct to 1 decimal place.

$q = 1.31$, correct to 2 decimal places.

Work out the lower bound of $\frac{p - q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the lower bound use LB of the top and UB of the bottom

Top: LB p − UB $q = 10.95 - 1.315 = 9.635$

Bottom: UB $r = 6.25$

$9.635 \div 6.25 = 1.54$

Answer: 1.54

8 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and $a > 0$.

The curve passes through the points (1, 8) and (4, 64).

Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 8$ and $k\,a^4 = 64$

Divide: $a^3 = \frac{64}{8} = 8$

$a = 2$

From the first point: $k \times 2 = 8$, so $k = 4$

Answer: $k = 4$, $a = 2$

9 Distance travelled

The diagram shows the graph of $s = 2t^3 - 18t^2 + 48t$, the displacement in metres of a particle P from O after t seconds.

P changes direction at A and B. Work out the total distance travelled in the first 5 seconds.

$v = \frac{ds}{dt} = 6t^2 - 36t + 48 = 0$ at $t = 2$ and $t = 4$ (A and B)

$s(0) = 0$, $s(2) = 40$, $s(4) = 32$, $s(5) = 40$

Distances: $|40 - 0| + |32 - 40| + |40 - 32|$

$= 40 + 8 + 8 = 56$

Answer: 56 m

10 A squared in terms of A and I

$A = \begin{pmatrix} 0 & -4 \\ -2 & -3 \end{pmatrix}$.

$A^2 = pA + qI$, where I is the 2 × 2 identity matrix.
Find the value of p and the value of q.

$A^2 = \begin{pmatrix} 8 & 12 \\ 6 & 17 \end{pmatrix}$

Top right entries: $12 = p \times (-4)$, so $p = -3$
Top left entries: $8 = -3 \times 0 + q$, so $q = 8$
Check bottom right: $-3 \times (-3) + 8 = 17$
Answer: $p = -3$, $q = 8$