Day 305 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

Differentiate $6x^4 + 6x^2 + 2x$ with respect to x.

Differentiate each term: $x^n \rightarrow nx^{n-1}$ and a constant $\rightarrow 0$

$6x^4 \rightarrow 24x^3$, $6x^2 \rightarrow 12x$, $2x \rightarrow 2$

$\frac{dy}{dx} = 24x^3 + 12x + 2$

Answer: $\mathbf{\frac{dy}{dx} = 24x^3 + 12x + 2}$

2 Time zones

A flight leaves London at 23:20 (London time).
It takes 12 hours 40 minutes.
When it is 12:00 in London it is 04:00 in Los Angeles.
At what time, in Los Angeles time, does the flight land? Say if it is on a different day.

Arrival in London time = 23:20 + 12 h 40 min = 12:00 the next day
Los Angeles is 8 hours behind London, so subtract 8 hours
04:00 the next day
Answer: 04:00 the next day

3 Weighted percentages

The ratio of men to women in a school is 3 : 1.
25% of the men and 55% of the women own a car.
What percentage of all of these students own a car?

Imagine 80 students in total: 60 men and 20 women
25% of 60 = 15 and 55% of 20 = 11
Together: 26 out of 80
26 ÷ 80 × 100 = 32.5%
Answer: 32.5%

4 Venn diagram: reading set notation

The Venn diagram shows information about 28 employees.
A: speak German B: speak Italian
(a) Find $n(A' \cup B')$.
(b) Find $n(A' \cap B)$.

(a) $n(A' \cup B') = 14 + 8 + 3 = 25$
(b) $n(A' \cap B) = 8$
Answer: (a) 25 (b) 8

5 Reverse percentage growth and decay

A laptop loses 10% of its value each year.
After 2 years it is worth £25,515.
Work out its value when it was new. (Calculator)

Yearly multiplier = 0.9
Original $\times 0.9^2 =$ £25,515
Original = £25,515 ÷ 0.9^2 = £31,500
Answer: £31,500

6 Tree diagrams and reverse conditional probability

The probability that it rains is $\frac{3}{4}$.

If it rains, the probability that Sam is late is $\frac{2}{3}$. If it does not rain, the probability is $\frac{1}{6}$.

(a) Work out the probability that Sam is late.
(b) Given that Sam is late, work out the probability that it rained.

(a) P(rain and late) = $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$; P(no rain and late) = $\frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$

P(late) = $\frac{1}{2} + \frac{1}{24} = \frac{13}{24}$

(b) P(rain | late) = $\frac{\frac{1}{2}}{\frac{13}{24}} = \frac{12}{13}$

Answer: (a) $\frac{13}{24}$ (b) $\frac{12}{13}$

7 Minimum surface area of a tank

A tank is a cuboid with a square base of side x metres and height h metres, and has a closed top. Its volume is 8 m^3.

(a) Show that the total outside surface area, A m^2, is $A = 2x^2 + \frac{32}{x}$.

(b) Find the minimum possible value of A, showing that it is a minimum.

$V = x^2h = 8$, so $h = \frac{8}{x^2}$

$A = 2x^2 + 4xh = 2x^2 + 4x \times \frac{8}{x^2} = 2x^2 + \frac{32}{x}$

$\frac{dA}{dx} = 4x - 32x^{-2} = 0$ gives $x^3 = 8$, so $x = 2$

$\frac{d^2A}{dx^2} = 4 + 64x^{-3} = 12 > 0$, so a minimum

$A = 24$

Answer: 24 m^2

8 Exact trigonometric values

Work out the exact value of sin 30° ÷ tan 45°
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\sin 30° \div \tan 45° = \frac{1}{2} \div 1$

$= \frac{1}{2}$

Answer: $\frac{1}{2}$

9 Perpendicular bisector

A is the point (6, −8) and B is the point (−2, 2).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{6 + (-2)}{2}, \frac{-8 + 2}{2}) = (2, -3)$

Gradient of AB = $\frac{2 - (-8)}{-2 - 6} = -\frac{5}{4}$

The perpendicular gradient is $-1 \div (-\frac{5}{4}) = \frac{4}{5}$

Using the midpoint: $y + 3 = \frac{4}{5}(x - 2)$

Multiply by 5 and rearrange: $-4x + 5y = -23$
Multiply by −1: $4x - 5y = 23$
Answer: $4x - 5y = 23$

10 Increasing for all x

$f(x) = x^3 + kx^2 + 75x + 1$, where k is a constant.
Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 75$
f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots
Discriminant: $(2k)^2 - 4 \times 3 \times 75 < 0$, so $4k^2 < 900$
$k^2 < 225$, so $-15 < k < 15$
Answer: $-15 < k < 15$