Day 304 **IGCSE 10-a-day: Model Solutions**

1 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 2) cm, DE = 6 cm and BC = 15 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 2} = \frac{6}{15}$
$15x = 6(2x + 2)$
$15x = 12x + 12$
$3x = 12$
$x = 4$
(b) AB = 2x + 2 = 10 cm
Answer: (a) x = 4 (b) 10 cm

2 Where two tangents meet

The curve C has equation $y = -x^2 + 2x - 5$.
The points A and B on C have x coordinates 0 and 3.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = -2x + 2$. At A: gradient 2, A = (0, −5)
Tangent at A: $y = 2x - 5$. At B: gradient −4, B = (3, −8)
Tangent at B: $y = -4x + 4$
Equate: $2x - 5 = -4x + 4$, so $x = \frac{3}{2}$ and $y = -2$

Answer: T $(\frac{3}{2}, -2)$

3 Tree diagrams without replacement

A bag contains 3 red counters, 6 blue counters and 3 green counters.
Two counters are taken at random without replacement.
Work out the probability that (a) at least one is green, (b) exactly one is green.

(a) P(at least one green) = 1 − P(none) = $1 - \frac{9}{12} \times \frac{8}{11} = \frac{5}{11}$
(b) P(exactly one green) = $\frac{3}{12} \times \frac{9}{11} + \frac{9}{12} \times \frac{3}{11} = \frac{9}{22}$
Answer: (a) $\frac{5}{11}$ (b) $\frac{9}{22}$

4 Image of a point under a graph transformation

The point P(−3, 9) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x + 3)
(b) $y = f(\frac{x}{2})$

(a) y = f(x + 3): x is decreased by 3, so P maps to (−6, 9)
(b) $y = f(\frac{x}{2})$: x is multiplied by 2, so P maps to (−6, 9)
Answer: (a) (−6, 9) (b) (−6, 9)

5 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by

$h = -3t^2 + 12t + 5$

(a) Work out the greatest height reached by the ball.

(b) Work out the velocity of the ball when t = 4. State whether it is moving up or down.

$\frac{dh}{dt} = -6t + 12 = 0$ gives $t = 2$

$\frac{d^2h}{dt^2} = -6 < 0$ so maximum. $h = 17$

When $t = 4$: $\frac{dh}{dt} = 12 - 6 \times 4 = -12$

Answer: (a) 17 m (b) −12 m/s, moving down

6 Where a line meets the axes

The line L has equation $8x - 15y = 120$.

L crosses the x-axis at the point A and the y-axis at the point B.

(a) Find the coordinates of A and B.

(b) Work out the area of triangle OAB, where O is the origin.

(a) At A, y = 0: $8x = 120$, so $x = 15$. A is (15, 0)

At B, x = 0: $-15y = 120$, so $y = -8$. B is (0, −8)

(b) OA = 15 and OB = 8, and angle AOB = 90°

Area $= \frac{1}{2} \times 15 \times 8 = 60$

Answer: (a) A(15, 0), B(0, −8) (b) 60

7 Writing inequalities from a situation

On a minibus there are x adults and y children.

There are at most 60 people. There are at least 3 children for every adult. There are at least 5 adults.

(a) Write down three inequalities for x and y.

(b) Find the greatest number of children on the minibus.

(a) At most 60 people: $x + y \leq 60$

At least 3 children for every adult: $y \geq 3x$

At least 5 adults: $x \geq 5$

(b) The most children occurs with the fewest adults: $x = 5$

$y \leq 60 - 5 = 55$ and $y \geq 15$, so $y = 55$ is allowed

Answer: (a) $x + y \leq 60$, $y \geq 3x$, $x \geq 5$ (b) 55

8 Bounds of area and perimeter

A rectangle has length 12 cm and width 14 cm.

Both measurements are correct to the nearest centimetre.

Evan says that the area of the rectangle could be exactly 185 cm^2.

Is Evan correct? You must show your working.

$11.5 \leq$ length < 12.5 and $13.5 \leq$ width < 14.5

Lower bound of area $= 11.5 \times 13.5 = 155.25$

Upper bound of area $= 12.5 \times 14.5 = 181.25$

So $155.25 \leq$ area < 181.25

185 is outside this interval, so Evan is not correct

Answer: No, 185 is outside the interval

9 Equations in disguise

Solve $x - 8\sqrt{x} + 12 = 0$

Let $u = \sqrt{x}$: $u^2 - 8u + 12 = 0$
Factorise: $(u - 6)(u - 2) = 0$, so $u = 6$ or $u = 2$
$\sqrt{x} = 6$ or $\sqrt{x} = 2$
$x = 36$ or $x = 4$
Answer: x = 4 or x = 36

10 Cylinder of fixed surface area

A closed cylinder (a tin) has radius r cm, height h cm and total surface area 24π cm^2.
(a) Show that its volume, V cm^3, is $V = 12\pi r - \pi r^3$.
(b) Find the maximum value of V in terms of π, and show that h = 2r when V is a maximum.

$2\pi r^2 + 2\pi rh = 24\pi$, so $h = \frac{12 - r^2}{r}$
$V = \pi r^2 h = \pi r(12 - r^2) = 12\pi r - \pi r^3$
$\frac{dV}{dr} = 12\pi - 3\pi r^2 = 0$, so $r^2 = 4$ and $r = 2$
$\frac{d^2V}{dr^2} = -6\pi r < 0$, so a maximum. $h = \frac{12 - 4}{2} = 4 = 2r$
$V = \pi \times 2^2 \times 4 = 16\pi$
Answer: 16π cm^3