**Day 303** **IGCSE 10-a-day: Model Solutions**

## 1 Reverse percentages

The price of a laptop is increased by 12%.
The new price is then increased by 40%.
The final price is £156.80.
Work out the original price of the laptop.

Multipliers: 1.12, 1.4
Overall multiplier = 1.12 × 1.4 = 1.568
Original = final ÷ multiplier = £156.80 ÷ 1.568
= £100
**Answer: £100**

## 2 Turning point of a quadratic by differentiation

$f(x) = -3x^2 + 18x + 4$
Use differentiation to find the coordinates of the turning point of the graph of y = f(x). State whether it is a maximum or a minimum.

$f'(x) = -6x + 18$
$f'(x) = 0$: $-6x + 18 = 0$, so $x = 3$
$f(3) = 31$
$f''(x) = -6$, which is negative, so it is a maximum
**Answer: (3, 31), maximum**

## 3 Weighted percentages

The ratio of boys to girls in a school is 4 : 1.
40% of the boys and 45% of the girls walk to school.
What percentage of all of these students walk to school?

Imagine 100 students in total: 80 boys and 20 girls
40% of 80 = 32 and 45% of 20 = 9
Together: 41 out of 100
41 ÷ 100 × 100 = 41%
**Answer: 41%**

## 4 Histogram with unequal class widths

The histogram shows the masses, in kg, of some parcels.
The bar for 65 to 75 represents 9 parcels.
Estimate the number of parcels with a value greater than 60.

Frequency = k × height × width. $9 = k \times 3 \times 10$ so $k = \frac{3}{10}$

Part of the 55 to 65 bar: $\frac{3}{10} \times 6 \times (65 - 60) = 9$

Bars above 65: 9
Total = 18
**Answer: 18**

## 5 Tangent to a factorised cubic

A curve has equation $y = (x - 1)(x - 2)(x - 3)$.
Find the equation of the tangent to the curve at the point where x = 2.

Expand: $y = x^3 - 6x^2 + 11x - 6$
$\frac{dy}{dx} = 3x^2 - 12x + 11$
At x = 2: y = 0 and gradient = −1
$y = -(x - 2)$
$y = -x + 2$
**Answer: $y = -x + 2$**

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## 6 Simplifying algebraic fractions

Simplify fully $\frac{2x^2 - 32}{x^2 + 4x - 32}$

Factorise the top: $2(x - 4)(x + 4)$
Factorise the bottom: $(x - 4)(x + 8)$
Cancel the common factor $(x - 4)$: $\frac{2(x + 4)}{x + 8}$

**Answer:** $\frac{\mathbf{2(x + 4)}}{\mathbf{x + 8}}$

## 7 Averages with algebra

Here are five numbers: $2x - 1$, $3x + 5$, $2x + 2$, $x + 2$, $2x - 3$.
The mean of the five numbers is 15.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = $(2x - 1) + (3x + 5) + (2x + 2) + (x + 2) + (2x - 3) = 15 \times 5 = 75$
$10x + 5 = 75$, so $10x = 70$ and $x = 7$
The numbers are 13, 26, 16, 9, 11
(b) Range = $26 - 9 = 17$
**Answer: (a) x = 7 (b) 17**

## 8 Show that lines are parallel

In triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$.
M is on OA and N is on OB, with OM : MA = ON : NB = 2 : 3.
(a) Show that MN is parallel to AB.
(b) Write down the ratio MN : AB.

(a) $\mathbf{OM} = \frac{2}{5}\mathbf{a}$ and $\mathbf{ON} = \frac{2}{5}\mathbf{b}$

$\mathbf{MN} = \mathbf{MO} + \mathbf{ON} = -\frac{2}{5}\mathbf{a} + \frac{2}{5}\mathbf{b} = \frac{2}{5}(\mathbf{b} - \mathbf{a})$

$\mathbf{b} - \mathbf{a} = \mathbf{AB}$, so $\mathbf{MN} = \frac{2}{5}\mathbf{AB}$: a multiple of $\mathbf{AB}$, so MN is parallel to AB

(b) MN : AB = 2 : 5
**Answer: (b) 2 : 5**

## 9 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $-2\mathbf{a} + 3\mathbf{b}$, OQ = $7\mathbf{a} + 9\mathbf{b}$ and OR = $13\mathbf{a} + 13\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $9\mathbf{a} + 6\mathbf{b}$
QR = OR − OQ = $6\mathbf{a} + 4\mathbf{b}$

QR = $\frac{2}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 2
**Answer: (b) 3 : 2**

## 10 Tangent perpendicular to a line

The tangent at the point P on the curve $y = -2x^2 + 2x - 4$ is perpendicular to the line $x + 6y = 6$.
Find the coordinates of P.

The line is $y = -\frac{1}{6}x + 1$, so its gradient is $-\frac{1}{6}$

The tangent has gradient 6

$\frac{dy}{dx} = -4x + 2 = 6$ so $x = -1$

$y = -8$

**Answer: $(-1, -8)$**