Day 302 **IGCSE 10-a-day: Model Solutions**

1 Gradient from a tangent

The diagram shows part of the curve $y = -2x^2 - x + 2$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)
(a) Use the two marked points on the tangent to work out its gradient.
(b) Use differentiation to find the x coordinate of P.

Gradient = $\frac{0-3}{2-(-1)} = -1$

$\frac{dy}{dx} = -4x - 1$

$-4x - 1 = -1$, so $x = 0$

Answer: (a) -1 (b) $x = 0$

2 Profit and loss

A shop buys a watch and adds a 60% mark-up to get the selling price.
In a sale the selling price is reduced by 20% to £230.40.
Work out how much the shop paid for a watch.

Overall multiplier = $1.6 \times 0.8 = 1.28$
Cost = £230.40 ÷ 1.28
= £180
Answer: £180

3 Histogram with unequal class widths

The histogram shows the times, in minutes, taken to finish a task.
The bar for 45 to 60 represents 6 people.
(a) Work out the frequency of the class 20 to 35.
(b) Work out the total frequency.

Frequency = k × height × width. For 45 to 60: $6 = k \times 2 \times 15$ so $k = \frac{1}{5}$

(a) 20 to 35: $\frac{1}{5} \times 7 \times 15 = 21$

(b) Frequencies: 21, 14, 6, 20
Total = 61
Answer: (a) 21 (b) 61

4 Bearings with the sine and cosine rules

A ship leaves a port P and travels 77 km on a bearing of 330° to Q. (Calculator)
It then travels 70 km on a bearing of 300° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is $330° - 180° = 150°$
Angle PQR = 150° (the difference between the bearings 150° and 300°)
$PR^2 = 77^2 + 70^2 - 2 \times 77 \times 70 \times \cos 150° = 20165$, so PR = 142.0 km

(b) $\cos(QPR) = \frac{77^2 + PR^2 - 70^2}{2 \times 77 \times PR}$, so angle QPR = 14.3°
R is anticlockwise of the direction PQ, so the bearing is $330° - 14° = 316°$
Answer: (a) 142.0 km (b) 316°

5 Quadratic inequalities

Solve $x^2 \leq 36$

Rearrange: $x^2 - 36 \leq 0$
Factorise: $(x + 6)(x - 6) \leq 0$
Critical values: $x = -6$ and $x = 6$
Sketch the U-shaped curve: the expression is negative or zero between the critical values
Answer: $-6 \leq x \leq 6$

6 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{2-0}{0-(-4)} = \frac{1}{2}$

Perpendicular gradient = $-1 \div \frac{1}{2} = -2$

M passes through B(−4, 0): $0 = (-2) \times (-4) + c$, so $c = -8$
$y = -2x - 8$
Answer: $y = -2x - 8$

7 Negative powers

Differentiate $-x^2 - \frac{2}{x^2}$ with respect to x.

Write negative powers as indices: $y = -x^2 - 2x^{-2}$
$-x^2 \to -2x$, $-2x^{-2} \to 4x^{-3}$
$\frac{dy}{dx} = -2x + 4x^{-3}$
Answer: $\frac{dy}{dx} = -2x + \frac{4}{x^3}$

8 Cumulative frequency graphs

The cumulative frequency graph gives information about the times of 100 students to complete a puzzle.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 100 had a time greater than 45.

(a) Median at cumulative frequency 50, so median = 30
Lower quartile at 25, so LQ = 20. Upper quartile at 75, so UQ = 40
Interquartile range = 40 − 20 = 20
(b) At 45, cumulative frequency = 84
Number greater = 100 − 84 = 16
Answer: (a) median 30, IQR 20 (b) 16

9 Tangent meets the curve again (cubic)

The tangent to the curve $y = x^3 + 2x^2 + 5x - 8$ at the point P where x = 0 meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 3x^2 + 4x + 5$, so at P the gradient is 5 and $y = -8$
Tangent: $y = 5x - 8$
Intersection: $x^3 + 2x^2 = 0$
x = 0 is a repeated root, so the roots are 0, 0 and x_Q
Sum of roots = −2, so $0 + x_Q = -2$ and $x_Q = -2$
y = −18
Answer: (−2, −18)

10 Tangent to a circle: proof

Show that the line $y = -3x - 10$ is a tangent to the circle $x^2 + y^2 = 10$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = -3x - 10$ into $x^2 + y^2 = 10$: $x^2 + (-3x - 10)^2 = 10$
Expand: $10x^2 + 60x + 90 = 0$
$b^2 - 4ac = 60^2 - 4 \times 10 \times 90 = 3600 - 3600 = 0$
$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent
$x = \frac{-60}{2 \times 10} = -3$, and $y = -3 \times (-3) - 10 = -1$. The point is (−3, −1)
Answer: (−3, −1)