Day 301 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $\frac{2x - y}{z}$

when $x = -1$, $y = -4$ and $z = -1$.

Substitute the values: $\frac{2 \times (-1) - (-4)}{-1}$

$= \frac{2}{-1}$

$= -2$

Answer: -2

2 Divide then differentiate

$y = \frac{-6x^3 + 4x^2 - 7x}{x}$

Find $\frac{dy}{dx}$.

Divide each term by x: $y = -6x^2 + 4x - 7$

$-6x^2 \rightarrow -12x$, $4x \rightarrow 4$

$\frac{dy}{dx} = -12x + 4$

Answer: $\frac{dy}{dx} = -12x + 4$

3 Capture and recapture

60 beetles in a field are caught, marked and put back.
Later, a second sample of 50 beetles is caught. 3 of these are marked.
(a) Work out an estimate for the number of beetles in a field.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 3 out of 50

This should match the proportion marked in the whole population: $\frac{3}{50} = \frac{60}{N}$

$N = 60 \times 50 \div 3 = 1000$

(b) No beetles are born, die or leave between the samples, and the beetles mix evenly

Answer: (a) 1000 (b) The population stays the same between samples

4 Maximum profit

A company makes and sells x thousand items. All amounts are in thousands of pounds.
The revenue is $R = x(21 - 2x)$ and the cost is $C = 6 + 5x$.
(a) Show that the profit $P = R - C$ is $P = -2x^2 + 16x - 6$.
(b) Find the number of items that must be sold for the profit to be a maximum, and this maximum profit.

$P = 21x - 2x^2 - (6 + 5x) = -2x^2 + 16x - 6$

$\frac{dP}{dx} = -4x + 16 = 0$ gives $x = 4$

$\frac{d^2P}{dx^2} = -4 < 0$, so a maximum

$P = 26$

Answer: 4000 items, maximum profit £26000

5 Area scale factor and determinant

Shape S has area 10 cm^2.

S is transformed by the matrix $\begin{pmatrix} 6 & 5 \\ 2 & 2 \end{pmatrix}$.

Work out the area of the image of S.

Area scale factor = |determinant| = $|6 \times 2 - 5 \times 2|$
$= |2| = 2$

Area of image = $2 \times 10 = 20$ cm^2

Answer: 20 cm^2

6 Average and instantaneous velocity

A particle moves in a straight line. Its displacement from O after t seconds is s metres, where
$s = t^2 + 4t + 8$

(a) Work out the average velocity of the particle between t = 2 and t = 4.

(b) Find the time at which the velocity equals this average velocity.

s(2) = 20 and s(4) = 40

Average velocity = $\frac{40 - 20}{4 - 2} = 10$

$v = \frac{ds}{dt} = 2t + 4$

$2t + 4 = 10$ gives $t = 3$

Answer: (a) 10 m/s (b) t = 3 s

7 Volume and surface area of composite solids

A solid is made from a cone of height 10 cm and a hemisphere. Both have radius 9 cm.
Work out the total volume of the solid. Give your answer in terms of π.

Cone: $\frac{1}{3} \times \pi \times 9^2 \times 10 = 270\pi$

Hemisphere: $\frac{2}{3} \times \pi \times 9^3 = 486\pi$

Total = $270\pi + 486\pi = 756\pi$ cm^3

Answer: 756π cm^3

8 Powers of standard form

Work out $(5 \times 10^{-4})^2$

Give your answer in standard form.

$(5 \times 10^{-4})^2 = 5^2 \times (10^{-4})^2$

$5^2 = 25$ and $(10^{-4})^2 = 10^{-8}$

$25 \times 10^{-8} = 2.5 \times 10^{-7}$

Answer: 2.5×10^{-7}

9 Sample space and conditional probability

A fair 6-sided dice and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.

(a) Work out the probability that the sum is a multiple of 3.

(b) Given that at least one number is 3, work out the probability that the sum is a multiple of 3.

(a) There are $6 \times 5 = 30$ equally likely outcomes, and 10 of them satisfy: the sum is a multiple of 3

Probability = $\frac{1}{3}$

(b) 10 outcomes satisfy: at least one number is 3. Of these, 2 also satisfy: the sum is a multiple of 3

Probability = $\frac{1}{5}$

Answer: (a) $\frac{1}{3}$ (b) $\frac{1}{5}$

10 Venn diagram with three sets: reading

The Venn diagram shows information about 47 shoppers.
A: bought fruit B: bought bread C: bought milk
(a) Find n(A ∪ C).
(b) Find n(A′ ∩ C).

(a) n(A ∪ C) = 7 + 6 + 3 + 9 + 6 + 3 = 34
(b) n(A′ ∩ C) = 6 + 3 = 9
Answer: (a) 34 (b) 9