Day 300 **IGCSE 10-a-day: Model Solutions**

1 Tangents parallel to a line

A curve has equation $y = x^3 + 3x^2 - 15x - 8$.
Find the equations of the two tangents to the curve that are parallel to the line $y = -6x - 2$.

Parallel lines have equal gradients: gradient $= -6$
$\frac{dy}{dx} = 3x^2 + 6x - 15 = -6$, so $3x^2 + 6x - 9 = 0$
$3(x + 3)(x - 1) = 0$ gives $x = -3$ or $x = 1$
At $x = -3$: $y = 37$, tangent $y - 37 = -6(x - (-3))$
At $x = 1$: $y = -19$, tangent $y - (-19) = -6(x - 1)$
Answer: $y = -6x + 19$ and $y = -6x - 13$

2 Inverse of a linear function

$f(x) = -4x - 4$.
(a) Find $f^{-1}(x)$.
(b) Find $f^{-1}(1)$.

(a) Let $y = -4x - 4$, then rearrange to make x the subject
$x = \frac{-4 - y}{4}$, so $f^{-1}(x) = \frac{-4 - x}{4}$
(b) $f^{-1}(1) = -\frac{5}{4}$
Answer: (a) $f^{-1}(x) = \frac{-4 - x}{4}$ (b) $-\frac{5}{4}$

3 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 2 : 4.
45% of the Year 10 students and 30% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 120 students in total: 40 Year 10 students and 80 Year 11 students
45% of 40 = 18 and 30% of 80 = 24
Together: 42 out of 120
$42 \div 120 \times 100 = 35\%$
Answer: 35%

4 Normal in the form ax + by + c = 0

The point A lies on the curve $y = x^2 - 5x - 7$ where $x = 0$.
Find the equation of the normal at A. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

$y = -7$ at $x = 0$ and gradient of curve $= -5$
Gradient of normal $= \frac{1}{5}$
$y + 7 = \frac{1}{5}x$
$y = \frac{1}{5}x - 7$
Multiply by 5: $x - 5y - 35 = 0$
Answer: $x - 5y - 35 = 0$

5 Laws of indices with algebra

Simplify fully $\frac{6x^4y^2 \times 9x^2y^3}{3x^4y^6}$
Give your answer in the form kx^ay^b

Numbers: $6 \times 9 \div 3 = 18$
Powers of x: $4 + 2 - 4 = 2$
Powers of y: $2 + 3 - 6 = -1$
$= 18x^2y^{-1}$
Answer: $18x^2y^{-1}$

6 Proof with indices

Simplify fully $\frac{2^{n+3} - 2^n}{7}$

$2^{n+3} - 2^n = 2^n(2^3 - 1) = 2^n \times 7$

Divide by 7: $\frac{7 \times 2^n}{7} = 2^n$

Answer: 2^n

7 Bounds in geometry

A rectangle has length 8.5 cm and width 4.4 cm, both correct to 1 decimal place.
Work out the upper bound of the perimeter of the rectangle.

Each length is given to 1 decimal place, so it can be up to 0.05 cm either side
Length is between 8.45 and 8.55, width is between 4.35 and 4.45
Perimeter = 2 × (8.55 + 4.45) = 26 cm
Answer: 26 cm

8 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 42.5.
Work out the value of x.

Mid-points: 10, 30, 50, 70, 90
Estimated mean = Σfm ÷ Σf, so (1670 + 70x) ÷ (49 + x) = 42.5
1670 + 70x = 42.5(49 + x) = 2082.5 + 42.5x
(70 − 42.5)x = 2082.5 − 1670
27.5x = 412.5, so x = 15
Answer: x = 15

9 Linear programming in context

A workshop makes x tables and y chairs each week.
Each table needs 1 hour of work and 6 metres of wood. Each chair needs 6 hours of work and 3 metres of wood.
There are 30 hours of work and 48 metres of wood available.
The profit is £7 for each table and £7 for each chair.
(a) Write down two inequalities, other than $x \geq 0$, $y \geq 0$. (b) Find the greatest profit.

(a) $x + 6y \leq 30$ and $6x + 3y \leq 48$
(b) Vertices of the feasible region: (0, 0), (0, 5), (6, 4), (8, 0)
Profit = 7x + 7y: 0, 35, 70, 56
Greatest profit = £70 with x = 6, y = 4
Answer: (a) $x + 6y \leq 30$, $6x + 3y \leq 48$ (b) £70

10 Tangent meets the curve again (cubic)

The tangent to the curve $y = x^3 - 4x^2 - 6x + 6$ at the point P where x = 1 meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 3x^2 - 8x - 6$, so at P the gradient is −11 and y = −3
Tangent: $y = -11x + 8$
Intersection: $x^3 - 4x^2 + 5x - 2 = 0$
x = 1 is a repeated root, so the roots are 1, 1 and x_Q
Sum of roots = 4, so $2 + x_Q = 4$ and $x_Q = 2$
y = −14
Answer: (2, −14)