Day 30 **IGCSE 10-a-day: Model Solutions**

1 Solving trigonometric equations

Solve 2 sin x + 1 = 0 for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\sin x = -\frac{1}{2}$

Related acute angle: $\sin^{-1}(\frac{1}{2}) = 30.0°$

sine is negative in the 3rd and 4th quadrants, so $x = 180° + 30.0°$ and $x = 360° - 30.0°$
$x = 210.0°$ and $x = 330.0°$
Answer: $x = 210.0°$ or $x = 330.0°$

2 Negative powers

$y = -3x^4 + \frac{2}{x^3}$

Find $\frac{dy}{dx}$.

Write negative powers as indices: $y = -3x^4 + 2x^{-3}$
$-3x^4 \rightarrow -12x^3$, $2x^{-3} \rightarrow -6x^{-4}$
$\frac{dy}{dx} = -12x^3 - 6x^{-4}$
Answer: $\frac{dy}{dx} = -12x^3 - \frac{6}{x^4}$

3 Find unknown constants in a function

f(x) = ax + b, where a and b are constants.
$f(4) = 9$ and $f^{-1}(7) = 3$.
Find the value of a and the value of b.

$f^{-1}(7) = 3$ means $f(3) = 7$
$4a + b = 9$ and $3a + b = 7$
Subtract: $a = 2$, so $a = 2$
$b = 9 - 2 \times 4 = 1$
Answer: $a = 2$, $b = 1$

4 Three-part ratios

Jamal, Kira and Beth share some stickers.
Jamal : Kira = 8 : 7
Kira : Beth = 8 : 4
(a) Work out the ratio Jamal : Kira : Beth.
(b) They share 444 stickers altogether. How many stickers does Beth get?

(a) Make the Kira part the same: 8 : 7 = 64 : 56 and 8 : 4 = 56 : 28
Jamal : Kira : Beth = 64 : 56 : 28 = 16 : 14 : 7
(b) Total parts = 37, so 1 part = 444 ÷ 37 = 12
Beth gets 7 × 12 = 84 stickers
Answer: (a) 16 : 14 : 7 (b) 84 stickers

5 Simplifying algebraic fractions

Simplify fully $\frac{3x^2 - 75}{x^2 - 6x + 5}$

Factorise the top: $3(x - 5)(x + 5)$
Factorise the bottom: $(x - 5)(x - 1)$
Cancel the common factor $(x - 5)$: $\frac{3(x + 5)}{x - 1}$

Answer: $\frac{3(x + 5)}{x - 1}$

6 Minimum gradient of a curve

A curve has equation $y = x^3 - 3x^2 + 5x - 4$.
Find the minimum value of the gradient of the curve, and the value of x at which it occurs.

$\frac{dy}{dx} = 3x^2 - 6x + 5$

The gradient is least when $\frac{d^2y}{dx^2} = 0$: $6x - 6 = 0$, so $x = 1$

$\frac{d^2y}{dx^2}$ increases through zero here (its derivative is $6 > 0$), so this is a minimum

$\frac{dy}{dx} = 2$ when $x = 1$

Answer: minimum gradient 2 when x = 1

7 Transformed curve from a diagram

The diagram shows the graph of $y = f(x)$ with a minimum point at (1, 3).
The dashed curve C is the image of $y = f(x)$ after a translation.
The minimum point of C is (4, 0).
Write down the equation of C in terms of f.

The minimum point has moved from (1, 3) to (4, 0)

Translation by $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

x moves right by 3: $f(x - 3)$. y moves down by 3: -3

Answer: $y = f(x - 3) - 3$

8 HCF and LCM

The HCF of two numbers is 10 and their LCM is 240.
The smaller number is 30.
Work out the larger number.

HCF × LCM = product of the two numbers
10 × 240 = 2400
Larger number = 2400 ÷ 30 = 80

Answer: 80

9 Curves that touch

The curves $y = 2x^2 - 2x + 1$ and $y = 3x^2 - 8x + 10$ touch at one point.
Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: $x^2 - 6x + 9 = 0$

That is $(x - 3)^2 = 0$ with a repeated root $x = 3$

$y = 13$, so the point is (3, 13)

$\frac{dy}{dx} = 4x - 2$ on the first curve: gradient 10

Tangent: $y = 10x - 17$

Answer: (3, 13), $y = 10x - 17$

10 Bounds of roots

The volume of a cube is 538 cm^3, correct to the nearest cm^3.
Work out the upper bound of the edge length of the cube.
Give your answer correct to 3 significant figures. (Calculator)

$537.5 \leq \text{volume} < 538.5$

Upper bound of edge = $\sqrt[3]{538.5}$
= 8.13571... = 8.14 (3 s.f.)

Answer: 8.14 cm