Day 3 **IGCSE 10-a-day: Model Solutions**

1 Gradient function from a given point

A curve has equation $y = -2x^3 - 9x^2 - 12x + 4$.
The point A $(-1, k)$ lies on the curve. Work out the value of k and the gradient of the curve at A.

Substitute $x = -1$ into the equation of the curve
$y = 9$ when $x = -1$, so $k = 9$
$\frac{dy}{dx} = -6x^2 - 18x - 12$
At A: gradient = 0
Answer: k = 9, gradient = 0

2 Equation of a circle

A and B are the points (6, 7) and (−6, 15).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{6 + (-6)}{2}, \frac{7 + 15}{2}) = (0, 11)$
$r^2 = (6 - 0)^2 + (7 - 11)^2 = 36 + 16 = 52$
$x^2 + (y - 11)^2 = 52$
Answer: $x^2 + (y - 11)^2 = 52$

3 Distance-time graphs

The graph shows Zane's cycle ride to a café and back.
(a) Work out Zane's speed on the way back.
(b) Work out Zane's average speed for the whole ride, including the stop.

(a) Return: 48 km in 4 hours, so speed = 48 ÷ 4 = 12 km/h
(b) Total distance = 2 × 48 = 96 km, total time = 7.5 hours
Average speed = 96 ÷ 7.5 = 12.8 km/h
Answer: (a) 12 km/h (b) 12.8 km/h

4 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the radius of the circle.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 48 ÷ 2 = 24 cm
$OA^2 = OM^2 + AM^2 = 10^2 + 24^2 = 676$
OA = 26 cm
(b) sin(angle AOM) = $\frac{24}{26}$
Angle AOM = 67.38°, so angle AOB = 2 × 67.38 = 134.8°
Answer: (a) 26 cm (b) 134.8°

5 Counting subsets

The set A has 8 elements.
(a) Write down the number of subsets of A.
(b) Write down the number of proper subsets of A.

(a) Each element is either in or out of a subset: $2^8 = 256$
(b) Remove the set A itself: 256 − 1 = 255
Answer: (a) 256 (b) 255

6 Show that a line is a tangent

The line $y = -4x + 4$ meets the curve $y = 2x^2 - 4x + 4$.
Show that the line is a tangent to the curve and find the coordinates of the point where they touch.

Equate: $2x^2 - 4x + 4 = -4x + 4$, so $2x^2 = 0$
Discriminant: $0^2 - 4 \times 2 \times 0 = 0 - 0 = 0$
One repeated root, so the line touches the curve at one point only: it is a tangent
$x = \frac{-0}{2 \times 2} = 0$ and $y = 4$
Answer: (0, 4)

7 Histogram with unequal class widths

The histogram shows the ages, in years, of some visitors.
The bar for 15 to 25 represents 30 visitors.
Estimate the number of visitors with a value greater than 16.

Frequency = k × height × width. $30 = k \times 3 \times 10$ so $k = 1$
Part of the 15 to 25 bar: $1 \times 3 \times (25 - 16) = 27$
Bars above 25: 60 + 140
Total = 227
Answer: 227

8 Bounds of roots

The area of a square is 183 cm^2, correct to the nearest cm^2.
Work out the upper bound of the side length of the square.
Give your answer correct to 3 significant figures. (Calculator)

$182.5 \leq$ area < 183.5
Upper bound of side = $\sqrt{183.5}$
= 13.5462... = 13.5 (3 s.f.)
Answer: 13.5 cm

9 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and $a > 0$.
The curve passes through the points (1, 12) and (3, 192).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 12$ and $k\,a^3 = 192$
Divide: $a^2 = \frac{192}{12} = 16$
$a = 4$ (positive, since $a > 0$)
From the first point: $k \times 4 = 12$, so $k = 3$
Answer: $k = 3$, $a = 4$

10 Increasing for all x

$f(x) = x^3 + kx^2 + 27x + 4$, where k is a constant.
Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 27$
f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots
Discriminant: $(2k)^2 - 4 \times 3 \times 27 < 0$, so $4k^2 < 324$
$k^2 < 81$, so $-9 < k < 9$
Answer: $-9 < k < 9$