Day 299 **IGCSE 10-a-day: Model Solutions**

1 Averages with algebra

Here are five numbers: $3x + 4$, $3x$, $x + 4$, $x - 1$, $2x - 2$.
The mean of the five numbers is 17.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = $(3x + 4) + (3x) + (x + 4) + (x - 1) + (2x - 2) = 17 \times 5 = 85$
$10x + 5 = 85$, so $10x = 80$ and $x = 8$
The numbers are 28, 24, 12, 7, 14
(b) Range = $28 - 7 = 21$
Answer: (a) x = 8 (b) 21

2 Minimum distance with 1/t

A particle P moves on a straight line. At time t seconds ($t > 0$), its distance from a point O is s metres, where
$s = 3t^2 + \frac{384}{t}$
Show that P is at its minimum distance from O when $t = 4$, and find this minimum distance.

$s = 3t^2 + 384t^{-1}$, so $v = \frac{ds}{dt} = 6t - 384t^{-2}$

$v = 0$: $6t = \frac{384}{t^2}$, so $t^3 = 64$ and $t = 4$

$\frac{d^2s}{dt^2} = 6 + \frac{768}{t^3}$, which is 18 when $t = 4$, so a minimum
$s = 144$
Answer: 144 m

3 Multiplying 2 by 2 matrices

$A = \begin{pmatrix} -3 & 0 \\ -1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ -2 & -1 \end{pmatrix}$.
(a) Work out AB.
(b) Work out BA.

(a) AB: row 1 is $(-3) \times 2 + 0 \times (-2)$ and $(-3) \times 0 + 0 \times (-1)$; row 2 is $(-1) \times 2 + (-2) \times (-2)$ and $(-1) \times 0 + (-2) \times (-1)$
$AB = \begin{pmatrix} -6 & 0 \\ 2 & 2 \end{pmatrix}$
(b) $BA = \begin{pmatrix} -6 & 0 \\ 7 & 2 \end{pmatrix}$
Answer: (a) AB $= \begin{pmatrix} \mathbf{-6} & \mathbf{0} \\ \mathbf{2} & \mathbf{2} \end{pmatrix}$ **(b) BA** $= \begin{pmatrix} \mathbf{-6} & \mathbf{0} \\ \mathbf{7} & \mathbf{2} \end{pmatrix}$

4 Equation of a tangent

A curve has equation $y = x^2 + 2x + 2$.
Find the equation of the tangent to the curve at the point where $x = -2$.

When $x = -2$, $y = 2$
$\frac{dy}{dx} = 2x + 2$, so the gradient at $x = -2$ is -2
$y - 2 = -2(x + 2)$
$y = -2x - 2$
Answer: y = −2x − 2

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5 Column vectors

$\mathbf{a} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$

Work out the exact length of the vector $\mathbf{a} + 2\mathbf{b}$. Give your answer as a simplified surd.

$\mathbf{a} + 2\mathbf{b} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} + 2\begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ -10 \end{pmatrix}$

Length $= \sqrt{(4)^2 + (-10)^2} = \sqrt{116}$

$= 2\sqrt{29}$

Answer: $2\sqrt{29}$

6 Show that dy/dx factorises

$y = (x + 1)^2(x + 3)$

Show that $\frac{dy}{dx} = (x + 1)(3x + 7)$.

Expand: $y = x^3 + 5x^2 + 7x + 3$

$\frac{dy}{dx} = 3x^2 + 10x + 7$

Factorise: $3x^2 + 10x + 7 = (x + 1)(3x + 7)$

Answer: $\frac{dy}{dx} = (x + 1)(3x + 7)$

7 Rationalising the denominator

Write $\frac{1 + \sqrt{2}}{2 + \sqrt{2}}$ in the form $p + q\sqrt{2}$

where p and q are rational numbers.

Multiply top and bottom by $(2 - \sqrt{2})$:

Numerator: $(1 + \sqrt{2})(2 - \sqrt{2}) = \sqrt{2}$

Denominator: $4 - 2 = 2$

$= \frac{\sqrt{2}}{2}$

Answer: $\frac{\sqrt{2}}{2}$

8 Line of best fit

The scatter graph shows age of car (x) and value (y).

The line of best fit passes through (2, 18) and (8, 9).

(a) Find its equation in the form $y = mx + c$.

(b) Explain what its gradient tells you.

(a) Gradient $m = \frac{9 - 18}{8 - 2} = \frac{-9}{6} = -1.5$

$y = -1.5x + c$: using (2, 18), $18 = -1.5 \times 2 + c$, so $c = 21$

$y = -1.5x + 21$

(b) For each extra year of age, the value of the car (in £100s) goes down by 1.5

Answer: (a) $y = -1.5x + 21$ (b) For each extra year of age, the value of the car (in £100s) goes down by 1.5

9 Line and curve intersections

Solve the simultaneous equations

$y = x^2 + x + 5$

$y = -4x - 1$

Substitute: $x^2 + x + 5 = -4x - 1$

Rearrange: $x^2 + 5x + 6 = 0$

Factorise: $(x + 2)(x + 3) = 0$, so $x = -2$ or $x = -3$

When $x = -2$: $y = -4 \times (-2) - 1 = 7$

When $x = -3$: $y = -4 \times (-3) - 1 = 11$

Answer: $x = -3$, $y = 11$ and $x = -2$, $y = 7$

10 Fibonacci-type sequences

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$.

$u_1 = 1$ and $u_2 = x$.

(a) Show that $u_5 = 3x + 2$.

(b) Given that $u_5 = 20$, find x and hence work out u_8.

(a) $u_3 = 1 + x$, $u_4 = u_3 + u_2 = 2x + 1$, $u_5 = u_4 + u_3 = 3x + 2$

(b) $3x + 2 = 20$ so $3x = 18$ and $x = 6$

Terms: 1, 6, 7, 13, 20, 33, 53, 86

$u_8 = 86$

Answer: $x = 6$, $u_8 = 86$