**Day 298** **IGCSE 10-a-day: Model Solutions**

## 1 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 2. N is on AB with AN : NB = 2 : 1.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Show that MN is parallel to OB.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{2}{3}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a}) = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

(b) OM = $\frac{1}{3}\mathbf{a}$

MN = ON − OM = $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} - \frac{1}{3}\mathbf{a} = \frac{2}{3}\mathbf{b}$

MN = $\frac{2}{3}\mathbf{b}$, which is a multiple of OB, so MN is parallel to OB

**Answer: (a) $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ (b) MN = $\frac{2}{3}\mathbf{b}$, parallel to OB**

## 2 f′(x) notation

f(x) = $5x^3 - 4x^2 + 8x$
(a) Find f′(x).
(b) Work out the value of f′(−3).

(a) $5x^3 \to 15x^2$, $-4x^2 \to -8x$, $8x \to 8$
f′(x) = $15x^2 - 8x + 8$
(b) f′(−3) = 167
**Answer: (a) f′(x) = $15x^2 - 8x + 8$ (b) 167**

## 3 Line of best fit

The scatter graph shows hours of revision (x) and test mark (y).
The line of best fit passes through (2, 8) and (6, 10).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient m = $\frac{10 - 8}{6 - 2} = \frac{2}{4} = 0.5$
y = 0.5x + c: using (2, 8), 8 = 0.5 × 2 + c, so c = 7
y = 0.5x + 7
(b) For each extra hour of revision, the test mark goes up by 0.5
**Answer: (a) y = 0.5x + 7 (b) For each extra hour of revision, the test mark goes up by 0.5**

## 4 Kinematics with roots

A particle moves in a straight line. After t seconds its displacement from O is s metres, where
s = $16t^{\frac{3}{2}} + t^2$ ($t \geq 0$)
Find the velocity and the acceleration of the particle when t = 16.

v = $\frac{ds}{dt} = 2t + 24t^{\frac{1}{2}}$
a = $\frac{dv}{dt} = 2 + 12t^{-\frac{1}{2}}$
When t = 16: v = 128 and a = 5
**Answer: v = 128 m/s, a = 5 m/s$^2$**

## 5 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 20 cm. (Calculator)
The outer radius is 6 cm and the inner radius is 4 cm.
The density of the metal is 8.9 g/cm$^3$.
Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal = $\pi \times 6^2 \times 20 - \pi \times 4^2 \times 20 = \pi(36 - 16) \times 20$
= 1256.6 cm$^3$
Mass = density × volume = 8.9 × 1256.6 = 11184 g
= 11.2 kg (3 s.f.)
**Answer: 11.2 kg**

## 6 Histogram with unequal class widths

The histogram shows the ages, in years, of some visitors.
The bar for 10 to 20 represents 20 visitors.
(a) Work out the frequency of the class 20 to 35.
(b) Work out the total frequency.

Frequency = k × height × width. For 10 to 20: 20 = k × 2 × 10 so k = 1
(a) 20 to 35: 1 × 2 × 15 = 30
(b) Frequencies: 20, 30, 20, 30
Total = 100
**Answer: (a) 30 (b) 100**

## 7 Tangent perpendicular to a line

The tangent at the point P on the curve $y = 2x^2 + 6x - 2$ is perpendicular to the line $x - 6y = -4$.
Find the coordinates of P.

The line is $y = \frac{1}{6}x + \frac{2}{3}$, so its gradient is $\frac{1}{6}$
The tangent has gradient $-6$
$\frac{dy}{dx} = 4x + 6 = -6$ so $x = -3$
$y = -2$
**Answer: $(-3, -2)$**

## 8 Variation: joint and combined

P is directly proportional to r and directly proportional to t.
When r = 4 and t = 4, P = 160.
Find P when r = 5 and t = 6.

P = krt
Substitute: 160 = k × 16 so k = 10
P = 10 × 30 = 300
**Answer: P = 300**

## 9 Equations with algebraic fractions

Solve $\frac{3}{x-2} + \frac{1}{x-3} = \frac{3}{2}$

Multiply every term by $(x - 2)(x - 3)$: $3(x - 3) + (x - 2) = \frac{3}{2} \times (x - 2)(x - 3)$
Multiply by 2 to clear the fraction: $6(x - 3) + 2(x - 2) = 3(x - 2)(x - 3)$
Expand and rearrange so that one side is 0: $3x^2 - 23x + 40 = 0$
Factorise: $(x - 5)(3x - 8) = 0$
$x = \frac{8}{3}$ or $x = 5$
**Answer: $x = \frac{8}{3}$ or $x = 5$**

## 10 Exact lengths and areas

A rectangle has area $(12 + 10\sqrt{2})$ cm$^2$ and length $(4 + 2\sqrt{2})$ cm.
Find the width of the rectangle.

Give your answer in the form $a + b\sqrt{2}$

Width = area ÷ length = $\dfrac{12 + 10\sqrt{2}}{4 + 2\sqrt{2}}$

Multiply top and bottom by $(4 - 2\sqrt{2})$:

Numerator: $(12 + 10\sqrt{2})(4 - 2\sqrt{2}) = 8 + 16\sqrt{2}$

Denominator: $16 - 8 = 8$

Width = $1 + 2\sqrt{2}$

**Answer: $1 + 2\sqrt{2}$ cm**