Day 297 **IGCSE 10-a-day: Model Solutions**

1 Find constants from a turning point

The curve $y = x^3 + ax^2 + bx - 5$ has stationary points at $x = -1$ and $x = 3$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$
$x = -1$: $3 + -2a + b = 0$. $x = 3$: $27 + 6a + b = 0$
Subtract the equations: $-24 + -8a = 0$, so $a = -3$
Then $b = -9$
Answer: a = −3, b = −9

2 Substitution

Work out the value of $(p - q)^2 - (p + q)^2$
when $p = -4$, $q = 4$ and $r = -2$.

Substitute the values: $(-4 - 4)^2 - (-4 + 4)^2$
$= 64 - 0$
$= 64$
Answer: 64

3 Square roots as powers

$y = 2x^{5/2} + 2x^{3/2} + 2x$
Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = 2x^{5/2} + 2x^{3/2} + 2x$
$2x^{5/2} \rightarrow 5x^{3/2}$, $2x^{3/2} \rightarrow 3x^{1/2}$, $2x \rightarrow 2$
$\frac{dy}{dx} = 5x^{3/2} + 3x^{1/2} + 2$
Answer: $\frac{dy}{dx} = 5x^{3/2} + 3x^{1/2} + 2$

4 Upper and lower bounds

a = 7.1 and b = 4.3,
both correct to 1 decimal place.
Work out the upper bound of a × b.

Each value is rounded, so it can be up to 0.05 above or below
$7.05 \leq a < 7.15$; $4.25 \leq b < 4.35$
Upper bound = 7.15 × 4.35 = 31.1025
Answer: 31.1025

5 Transformation by a matrix: image and description

$M = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.
(a) Find the coordinates of the image of the point (−3, 4) under the transformation represented by M.
(b) Describe fully the single transformation represented by M.

(a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$
The image is (3, −4)
(b) The first column is where (1, 0) goes and the second column is where (0, 1) goes
M represents a rotation of 180° about the origin
Answer: (a) (3, −4) (b) Rotation of 180° about the origin

6 Combined transformations

The point P(−5, 3) is rotated 90° clockwise about the origin.
The image is then reflected in the line y = x.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (3, 5)
After the second transformation: P'' = (5, 3)
(b) Follow a general point (x, y) through both transformations: (x, y) → (y, −x) → (−x, y)
The single mapping is (x, y) → (−x, y)
This is a reflection in the y-axis
Answer: (a) (5, 3) (b) Reflection in the y-axis

7 Minimum gradient of a curve

A curve has equation $y = 2x^3 - 5x - 4$.
Find the minimum value of the gradient of the curve, and the value of x at which it occurs.

$\frac{dy}{dx} = 6x^2 - 5$

The gradient is least when $\frac{d^2y}{dx^2} = 0$: $12x = 0$, so $x = 0$

$\frac{d^2y}{dx^2}$ increases through zero here (its derivative is $12 > 0$), so this is a minimum

$\frac{dy}{dx} = -5$ when $x = 0$

Answer: minimum gradient −5 when x = 0

8 Geometric sequences

The 3rd term of a geometric sequence is 128.
The 5th term of the sequence is 512.
All the terms of the sequence are positive.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^2 = 512 \div 128 = 4$
$r = \sqrt{4} = 2$ (positive, since all terms are positive)
(b) The 3rd term is $a \times r^2$: $a \times 4 = 128$
$a = 128 \div 4 = 32$
Answer: (a) r = 2 (b) 32

9 Bounds of a compound expression

p = 14.3 and r = 4.9, each correct to 1 decimal place.
q = 1.34, correct to 2 decimal places.

Work out the upper bound of $\frac{p - q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the upper bound use UB of the top and LB of the bottom
Top: UB p − LB q = 14.35 − 1.335 = 13.015
Bottom: LB r = 4.85
13.015 ÷ 4.85 = 2.68
Answer: 2.68

10 Probability and quadratic equations

A bag contains some beads. 7 of them are blue and the rest are black.
Two beads are taken at random without replacement.

The probability that both are blue is $\frac{21}{55}$.

Work out how many beads are black.

Let there be n beads altogether

P(both blue) = $\frac{7}{n} \times \frac{6}{n-1} = \frac{42}{n(n-1)}$

$\frac{42}{n(n-1)} = \frac{21}{55}$, so $n(n-1) = 110$, giving $n^2 - n - 110 = 0$

$(n-11)(n+10) = 0$, so $n = 11$ (n cannot be negative)

Number of beads that are black = $11 - 7 = 4$

Answer: 4